Rail degradation predication: Melbourne tram system case study

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Abstract

Nowadays, tram as an accessible and convenient mode of public transport is implemented and used in different cities. Due to high frequency of accelerations and decelerations along their routes and sharing the route with other vehicles, the rate of degradation of tram tracks (light rail) is different from the degradation rate of train tracks (heavy rail). In this paper, track gauge deviation as an indicator of irregularities on the rail-wheel contact surface is used for developing the track degradation model. Data set used in this study includes the curve sections of Melbourne tram network and divided into repaired and unreppaired segments. For data analysis more than 13 km of curved tracks are examined. Annual tonnage, previous gauge deviation and track structural properties like track surface, rail support, rail profile and gauge deviation are considered as the influencing variables on the future gauge deviation. Two different models including a regression model and an Artificial Neural Networks (ANN) model have been developed for predicting tram track gauge deviation. According to the results, the performances of the regression models are not very different from the ANN models. The determination coefficients of the selected models are approximately 0.8 and higher.

Keywords: Degradation; Tram Track; Gauge; Artificial Neural Network; Regression

1. Introduction

Today, tram as an energy-efficient and non-polluting mode of public transport is implemented and used in different places to connect suburbs, cities and even countries (Ahac & Lakušić 2017). Based on reviewing successful implementations of trams systems in different cities, it can be expressed that comparing to other modes of public transport, this mode has some advantages. For instance, the tram tracks are mostly not grade-separated and most of vehicles are low-floor so boarding and alighting is easier and faster for passenger with disabilities and older people (Naegeli et al. 2012). In addition, as the trams share the road with other vehicles, trams have become as one of the accessible modes of transportation (Arvidsson & Browne 2013). Furthermore like other rail systems, producing no exhaust emission and having higher passenger capacity (comparing with conventional busses) can make them more popular among the commuters and public transport users (Dincer et al. 2015).

As the demand for using trams is increased, tram infrastructure should bear more loads. In other words, more frequencies of tram services or having larger weights on the rail tracks can lead to higher degradation rates. In railway infrastructure, the rate of mechanical degradation is not quick but this process can lead to massive failure with catastrophic human casualties and massive financial lose if necessary maintenance actions are not taken (Soleimanmeigouni and Ahmadi 2016). To keep the tram services safe, reliable and comfortable, having efficient maintenance policies are essential (Ahac and Lakušić 2015). In this regard, the European
countries have planned to invest more than 20 billion EUR per annum on maintenance and renewal of their railway infrastructure systems (Lidén 2015). In Australia, PTV (Public Transport Victoria) between 2014 and 2015 spent more than 450 million AUD for the maintenance and renewal activities of different type of rail systems including metropolitan, tram and regional networks (PTV 2015).

In this context, railway infrastructure maintenance management systems are designed to optimise and implement maintenance and renewal activities. The main task of such systems is to determine when and how to conduct maintenance activities. In addition, due to budget constraints, the way resources and funds are allocated must be optimised (Yousefikia et al. 2014). Railway infrastructure maintenance management systems cover different activities. The major activities which should be carried out correctly are railway monitoring and inspection, railway track degradation prediction modelling and development of short/long term maintenance strategies (Santos et al. 2015). It is notable that, prediction modelling of track degradation is the fundamental prerequisite for establishment of efficient and cost-effective maintenance strategies of a tram system. It is clear that without forecasting the future condition of rail tracks, designing and providing preventive maintenance strategies are impossible (Andrews 2012).

The aim of this research is to develop models to predict the tram track degradation based on the condition of rail tracks in the past years. It must be taken into account that several studies have been conducted in the field of heavy rail track degradation modelling but few studies have been carried out in tram track degradation modelling.

This paper is structured as follows. The relevant literature on degradation prediction modelling will be explained in the second section. The date set and area of the study which have been applied in this research will be presented in the third section, followed by the model development afterwards. Then, the results will be presented and discussed. The final section will provide the conclusions of this research and suggests directions for future research.

2. Literature review

Track degradation models combine statistical methods and engineering techniques to bring an equation which can be used to predict the future condition of rail tracks by considering the influencing parameters on degradation of rail tracks. By examining the railway literature, the degradation models can be categorised into three main categories including mechanistic models, statistical models and Artificial Intelligence (AI) models (Figure 1).

![Figure 1: Degradation models](image)

**2.1. Mechanistic models**

Mechanistic models are considered as the primary and traditional models which are employed to forecast the level of degradation of railway tracks. These types of models are based on mechanical characteristics of track components which can result in degradation. Sato (1995) conducted a research to assess the track deterioration due to the ballast settlement under train repeated loading passage. For carrying out this study, Tokaido Shinkansen rail tracks in Japan were examined. In this study, ballast settlement as an indicator of track degradation was considered as the dependent variable and number of loading, vertical acceleration to initiate slip and ballast acceleration were considered as the independent variables. The following equation is provided to predict the settlement of a tamped track under repeated loading by train passage:
\[ y = \gamma (1 - e^{-\alpha x}) + \beta x \]  

(1)

Where, \( y \) represents the ballast settlement, \( x \) represents the repeated number of loading carried by the track, \( \alpha \) represents the vertical acceleration required to initiate slip, \( \beta \) represents ballast acceleration and sleeper pressure and \( \gamma \) is a constant coefficient correspondent to the initial compacting of the ballast material.

The technical university of Munich (Demharter, 1982) has conducted a series of experiment under satisfactory controlled laboratory to calculate the rate of settlement as the dependent variable. Ballast pressure, pre-loading period and the total number of passing axles are independent variables. In this research the rate of settlement was calculated by the equation shown as follows:

\[ s = a \times p \times \ln N + b \times p^{1.21} \times \ln N \]  

(2)

Where, \( s \) represents the settlement rate, \( p \) represents the ballast pressure and could be calculated through the Zimmermann method. \( N \) denotes a pre-loading period in addition to the first passing axels. \( N \) in the second term is the total number of passing axles. The parameters \( a \) and \( b \) are constant coefficients.

### 2.2. Statistical models

A statistical degradation model as a type of mathematical model is based on input and output variables. In statistical models, having sufficient historical data records about rail tracks is essential. Statistical models can be divided into deterministic models, stochastic model and probabilistic models (Figure 2). Different practices have been done in track degradation modelling by employing statistical models.

![Statistical models](image)

**Figure 2: Statistical models**

Westgeest, Dekker, & Fischer, (2012) conducted a linear regression model to predict the effective contributors to the track deterioration progress and the volume of maintenance required over a long period of time. In this research, KPI (Key Performance Indicator) as a combination of track geometry parameters (including longitudinal levelling, horizontal alignment, cross-level, twist and gauge) was considered as the dependent variable. The segments with maintenance were not included. Tamping, passing tonnage, soil type, sleeper type and closeness to switches are considered as the independent variables. According to the result of the analysis, segments with switches have degradation rate faster than others. Segments contain concrete sleepers degrade slower than segments with hardwood sleepers. Subsoil clay has a little negative influence on degradation and on the contrary passing tonnage has a positive influence on the degradation value.

Andrade and Teixeira (2012) developed a Bayesian model in Lisbon-Oporto line to assess rail track degradation through its life cycle. The purpose of this study was to investigate the evolution of uncertainty associated with track degradation parameters over the rail track life-cycle. In this study standard deviation of longitudinal levelling defects was considered as the main dependent parameter and primary standard deviation measured after tamping operations or renewal (mm), the rate of deterioration (mm/100 MGT) and the accumulated tonnage since tamping operations or renewal (100 MGT) were considered as independent variables. The results of the study
showed that at the design stage, the uncertainty associated with degradation rates is extremely large, but it reduces dramatically as more track inspection data is gathered.

Ahac and Lakušić (2015) applied multi-stage regression models on different Zagreb tram track segments in order to develop a maintenance-planning framework. Multistage regression model is a type of linear regression model which has the capability to cope with different stages of degradation phases or cope with the degradation process between two restorations or consecutive maintenances. For developing the model, gauge deviation difference value was considered as the dependent variable and cumulative exploitation intensity which is the result of multiplication of daily gross mass of trams with passengers (MGT) and the total number of exploitation days were considered as the independent variables. Based on the findings of this research, gauge degradation can be split into three main phases including moderate increase in tram track gauge taken place in the first phase which is followed by faster growth in gauge degradation in the second phase. In the third phase and for values above 45 MGT the model does not provide the gauge degradation accurately.

2.3. Artificial Intelligence models

AI techniques are development and activity systems that reproduce the cognitive skills of human experts to assist users facing intricate decision-making processes (Ozden 2017). Various AI models such as Artificial Neural Networks (ANN) and Decision Support Systems (DSS) have been used in track degradation modelling.

Sadeghi and Askarinejad (2012) developed an ANN model to evaluate railway track quality condition. They examined the possibility of having correlation between track geometrical defects and track structural problems. In their study, the network input were standard deviations of track geometry data (gauge, profile, alignment and twist) and the output variable is the prediction of defect density (defect density is defined as the ratio of the amount of defected length of a railway segment to the total length of the segment) of track structural components. The best performance of the model was achieved when the ANN model with 25 neurons in the hidden layer was developed and standard deviations of profile, alignment and twist are considered as the input variables.

Guler (2013) elaborated a DSS to perform railway track maintenance and renewal management in Turkish State Railway. The proposed decision support system was designed by conducting comprehensive literature reviews and interviewing with domain experts. The parameters used in their model include ballast, tamping history, type of sleep, gauge value, track class, number of trains, age of rails, speeds and cost analysis. Different M&R (maintenance and renewal) operations were addressed in this study such as ballast renewal, sleeper renewal, rail grinding, rail renewal and rail lubrication. Four different levels of M&R actions were introduced including do nothing, regular maintenance actions, corrective maintenance and renewal. The proposed DSS has a capability to renew itself by changing or including new rules.

3. Study area and data set

With a total of 25 routes and more than 1700 stops, Melbourne tram network is consisted of 250 km of double tracks which is considered as one of the largest tram networks in the world. By employing 450 in-service tram cars, this mode of public transport has provided more than 203 million journeys in 2016 which demonstrates the 12 percent growth compared to 2015. Along with the increase in Melbourne tram patronage, it has been observed that the expenses related to tram infrastructure such as tracks, switches and crossings has increased gradually and continuously. It is clear that there is a direct relationship between the number of passengers travelling by trams and the infrastructure expenses as the increase in trips and tram frequencies can result in more pressure and stress on the infrastructure components. In this study, data set of the tram is provided by the Yarra Trams which is the operator of
Melbourne tram network. Current data set composed of different section types including curves, straight sections, H-crossing and crossovers. There is a wide range of parameters covered in the data set but the major parameters are deviation of track geometry parameters (e.g. gauge, twist and longitudinal levelling) at different years, curve radius, annual tonnage in Million Gross Tone (MGT), track surface (in tram routes track surface are mostly categorised into asphalt and concrete surfaces), rail profile (the cross sectional shape of a rail which represented by kilogram per metre), rail type (Grooved and T-shapes), rail support (or rail ties categorised into concrete and steel sleepers), location of routes and track installation date. The data was collected from 2009 to 2015. It is noted that this study only focuses on curve sections. In total, more than 13 km of curved tracks are examined. Tracks are divided into 20 m length segments which have homogeneous characteristics.

4. Model development

Tram track gauge is considered as one of the most important track geometry parameters. Rail track degradation, which appears as gradual increase in gauge deviation from prescribed values during track exploitation, can lead to poor passenger ride quality, safety issues and higher maintenance costs. In this study, predicting degradation of track gauge as an indicator of degradation of whole tram track infrastructure has been targeted. In this research in order to have a broader picture of track degradation modelling, based on the applied techniques (regression or ANN) and the condition of track segments (repaired or unrepaird) totally four different models have been developed and analysed.

4.1 Regression model

For developing regression models (including models for repaired segments and unrepaird segments), SPSS statistics software is used. Based on reviewing the relevant researches, the input variables are selected including track gauge deviation in previous year (\(G_{t-1}\)), MGT, rail support (\(R_S\)), track surface (\(T_S\)), rail profile (\(R_P\)) and rail type (\(R_T\)). The output variable is track gauge in current year (\(G_t\)). MGT and \(G_{t-1}\) parameters are continuous parameters but the other parameters including rail support, rail profile, track surface and rail type are categorical variables. For developing a regression model involving categorical variables, dummy variables which take 0 and 1 must be defined to represent categorical variables. The list of dummy variables which have been used to develop a primary regression model has been represented in Table 1. The results of the proposed regression models are presented and discussed in the fifth section.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Converted dummy variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rail support</td>
<td>Concrete=1 and Steel sleepers=0</td>
</tr>
<tr>
<td>Rail type</td>
<td>Grooved=1 and T-shapes=0</td>
</tr>
<tr>
<td>Track surface</td>
<td>Asphalt=1 and Concrete=0</td>
</tr>
<tr>
<td>Rail profile</td>
<td>42 kg, 57 kg, 60 kg, 96 lb and others</td>
</tr>
</tbody>
</table>

According to the literature, different methods exist to assess the performance of the proposed models. In order to obtain an indication of goodness of fit between the observed and predicted values the coefficient of determination (\(R^2\)) has been used. Beside \(R^2\) which provides useful information on the goodness of fit of a model, the Mean Squared Error (MSE) has been used to evaluate the performance of the models in this research (Eq. 3). This error is a suitable indicator for the quality of the adjustment of the model.
Where, MSE is the mean-squared error, N represents the number of samples, $y_{predicted}$ is the value predicted by the model and $y_{actual}$ is the actual value.

### 4.2 ANN model

ANN as a branch of AI techniques is consisted of a number of independent interconnected neurons which can communicate with each other via weighted connections. In ANN models, a neuron can produce an outcome using values directly derived from other neurons. In this study, Matlab as a numerical analysis software was used. Similar to the relevant literatures (Sadeghi & Askarinejad 2012; Tedeschi & Benedetto 2016; Ferrer & Ruiz 2014; Hakan Guler 2013) the widely used multilayer feed-forward network is applied. In this model type, the neurons are arranged in a layered architecture and the signals are conveyed layer by layer in a forward direction style through the network. The mathematical mechanism of a neuron in ANN model can be formulated as follows:

$$O_i = A \left( \sum_{j=1}^{n} \omega_{ji} \cdot I_j + B_i \right)$$

(4)

Where, $A$ is the transfer (activation) function, $\omega_{ji}$ is the synaptic weight of the $j^{th}$ in-edge, $I_j$ is the in-edge, $I_j$ is the input labelled with the $j^{th}$ in-edge and $B_i$ is the bias associated with the $i^{th}$ neuron. The error back propagation algorithm as supervised learning is used for the purpose of training data set. This type of algorithm is a common procedure of training. In this algorithm the error signals deriving from the difference between the actual and expected outputs are back-propagated from the output layer to the previous layers to update the weights of connections and biases were adjusted repeatedly based on the computed errors of the network (Pazos et al. 2008). In this study, the 70 percent of data set was assigned for training the networks and the remaining data was dedicated to test the performance of the networks. The testing data were independent of training data. A four layered network (Figure 3) has been considered in this study which contains an input layer, two hidden layers and an output layer. A tan-sigmoid function (TANSIG) was used for the hidden layers and a linear transfer function (PURELIN) was used for the output layer.

![Figure 3: A typical architecture of a four-layered ANN model](image-url)
Based on the findings from the regression model, two different ANN models with the combination of explanatory variables which mentioned before have been developed for repaired segments and unrepaird segments. For each model different numbers of neurons in hidden layers were considered (7, 10, 15, 20 and 25). The results of the proposed ANN models are presented and discussed in the following section.

5. Results and discussion

Based on different model developments, two linear multiple regression models which have obtained good results in prediction of track gauge (Gt) for repaired segments and unrepaired segments are selected and the results of the regression analysis are represented in Table 2 and Table 3. In these regression models, input variables which were not useful in estimating the dependent variables (p-value is greater than 0.05) were removed and then the regression models were redeveloped based on the remaining parameters.

According to the results of the model for the repaired segments (Table 2), Gt-1, Ts, Rs and MGT are significant at a 95 percent confidence level to estimate the current gauge. But among these parameters the impact of rail support on gauge degradation is higher than others. MGT has the lowest impact on gauge degradation and due to its low coefficient can be removed from the model. Although the effect of MGT on gauge deviation is negligible but it must be noted that its impact lies in previous gauge deviation. Previous gauge deviation with positive coefficient has a clear correlation with current gauge deviation. The coefficient of track surface and rail support are negative and with regard to their definition (rail support: concrete=1 and steel sleepers=0 & track surface: asphalt=1 and concrete=0), it can be expressed that concrete sleeper can decrease the rate of degradation compared to steel sleeper. Also according to this table, the rate of degradation in repaired track segments surfaced with asphalt will be lower compared to tracks surfaced with concrete. Adjusted R squared is about 0.8 and the value of MSE is 1.311 which means that results of the model are acceptable.

Table 2: The result of the regression model for the repaired segments

<table>
<thead>
<tr>
<th>Variables</th>
<th>Coefficients</th>
<th>Std. Error</th>
<th>t-Statistics</th>
<th>Sig.</th>
<th>Adjusted R squared</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>5.343</td>
<td>0.769</td>
<td>6.948</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gt−1</td>
<td>1.007</td>
<td>0.044</td>
<td>22.981</td>
<td>0.000</td>
<td>0.779</td>
<td>1.311</td>
</tr>
<tr>
<td>Ts</td>
<td>-1.152</td>
<td>0.250</td>
<td>-4.599</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rs</td>
<td>-3.203</td>
<td>0.582</td>
<td>-5.499</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MGT</td>
<td>-1.843E-07</td>
<td>0.000</td>
<td>-3.368</td>
<td>0.001</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

According to Table 3, likewise the previous regression model, Gt-1, Ts and MGT are significant at a 95 percent confidence level to estimate the current gauge in unrepaired track segments. Again, MGT has the lowest impact on gauge degradation and due to its coefficient can be removed from the model. Previous gauge deviation with positive coefficient has a clear correlation with current gauge deviation. Contrary to the previous model but with lower coefficient, track segments surfaced with asphalt have higher degradation rate to those covered by concrete. Adjusted R squared is 0.84 and the value of MSE is 1.501 which means that results of the model are satisfying.
Table 3: The result of the regression model for the unrepaired segments

<table>
<thead>
<tr>
<th>Variables</th>
<th>Coefficients</th>
<th>Std. Error</th>
<th>t-Statistics</th>
<th>Sig.</th>
<th>Adjusted R squared</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.741</td>
<td>0.173</td>
<td>4.285</td>
<td>0.000</td>
<td>0.840</td>
<td>1.501</td>
</tr>
<tr>
<td>G_{t-1}</td>
<td>1.115</td>
<td>0.040</td>
<td>28.084</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T_s</td>
<td>0.975</td>
<td>0.288</td>
<td>3.386</td>
<td>0.001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MGT</td>
<td>-1.425E-07</td>
<td>0.000</td>
<td>-3.083</td>
<td>0.002</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In Table 4 and Table 5 the results of ANN models both for repaired segments and unrepaired segments are shown. These models are based on the explanatory variables mentioned in previous regression models. In spite of the fact that ANN models consisting with a large number of hidden neurons are more likely to produce results with less training errors, as these models may not succeed to bring a generalized solution suitable to different sample of the problem, they sometimes lead to higher validation errors and consequently lower coefficient of determination. In this concept, the values of performance indicators of the models, $R^2$ and MSE have been changed based on the number of neurons in the proposed models. The best result for the repaired segments is achieved when the number of neurons in the first hidden layer is 10 and the number of neurons in the second hidden layer is 7 as $R^2$ equals 0.786 and MSE value is 1.387. Respectively, the best result for the unrepaired segments is achieved when the number of neurons in the first hidden layer is 15 and the number of neurons in the second hidden layer is 10 as $R^2$ equals 0.872 and MSE value is 1.433.

Table 4: The results of ANN model for the repaired segments

<table>
<thead>
<tr>
<th>No. of neurons in the first hidden layer</th>
<th>neurons in the second hidden layer</th>
<th>Adjusted $R^2$</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>7</td>
<td>0.786</td>
<td>1.387</td>
</tr>
<tr>
<td>15</td>
<td>10</td>
<td>0.810</td>
<td>1.810</td>
</tr>
<tr>
<td>20</td>
<td>15</td>
<td>0.748</td>
<td>1.870</td>
</tr>
<tr>
<td>25</td>
<td>20</td>
<td>0.466</td>
<td>2.177</td>
</tr>
</tbody>
</table>

Table 5: The results of ANN models for the unrepaired segments

<table>
<thead>
<tr>
<th>No. of neurons in the first hidden layer</th>
<th>neurons in the second hidden layer</th>
<th>Adjusted $R^2$</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>7</td>
<td>0.664</td>
<td>5.42</td>
</tr>
<tr>
<td>15</td>
<td>10</td>
<td>0.872</td>
<td>1.433</td>
</tr>
<tr>
<td>20</td>
<td>15</td>
<td>0.697</td>
<td>6.240</td>
</tr>
<tr>
<td>25</td>
<td>20</td>
<td>0.160</td>
<td>11.027</td>
</tr>
</tbody>
</table>

These results are consistent with the literature on rail track degradation models which show that geometry condition of a rail track during its life time is strongly related to its primary level.
Also the literature support the results of this research about the impact of track surface and rail support (sleeper) on rail track degradation.

6. Conclusion and future research directions

Track gauge is one of the important geometric parameters which can be used as an indicator for ride comfort and safety in tram system. Prediction of the future condition of gauge can help tram operators in establishing tram maintenance management systems to reduce maintenance cost and improve service quality. In this research, the data sets of Melbourne tram track for different years have been examined. To predict the gauge degradation firstly, two regression models based on repaired segments and un repaired segments data have been developed. Then by applying ANN method and based on the combination of explanatory variables used in development of the regression models, two ANN models were created. Table 6 shows the comparison of the results using the regression and ANN methods. According to this table, the performance of regression model in prediction of gauge deviation for repaired segments compared to the ANN model is slightly better, while for un repaired segments the inverse relationship is happened.

Table 6: The comparison of the results.

<table>
<thead>
<tr>
<th>Model</th>
<th>Type</th>
<th>Adjusted R squared</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Repaired segments</td>
<td>Regression</td>
<td>0.799</td>
<td>1.311</td>
</tr>
<tr>
<td></td>
<td>ANN</td>
<td>0.786</td>
<td>1.387</td>
</tr>
<tr>
<td>Unrepaired segments</td>
<td>Regression</td>
<td>0.840</td>
<td>1.501</td>
</tr>
<tr>
<td></td>
<td>ANN</td>
<td>0.872</td>
<td>1.433</td>
</tr>
</tbody>
</table>

Developing degradation models for straight sections, H-crossings and crossovers can be a future direction for this research. Furthermore, parallel with the AI techniques and regression models, using other statistical methods such as stochastic models for predicting tram track degradation and comparing them with the current models can improve the overall judgment about the deterioration process in track segments.

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