Arterial travel time estimation: Revisiting the classical procedure

Ashish Bhaskar¹, Edward Chung², André-Gilles Dumont³

¹,²Faculty of Built Environment and Engineering Queensland University of Technology, 2 George St, Brisbane, QLD, 4001, Australia
³Ecole Polytechnique Fédérale De Lausanne, Lausanne 1015, Switzerland

Email for correspondence: ashish.bhaskar@qut.edu.au

Abstract

Travel time is an important network performance measure and it quantifies congestion in a manner easily understood by all transport users. In urban networks, travel time estimation is challenging due to number of reasons such as, fluctuations in traffic flow due to traffic signals, significant flow to/from mid link sinks/sources, etc. The classical analytical procedure utilizes cumulative plots at upstream and downstream locations for estimating travel time between the two locations. In this paper, we discuss about the issues and challenges with classical analytical procedure such as its vulnerability to non conservation of flow between the two locations. The complexity with respect to exit movement specific travel time is discussed. Recently, we have developed a methodology utilising classical procedure to estimate average travel time and its statistic on urban links (Bhaskar, Chung et al. 2010). Where, detector, signal and probe vehicle data is fused. In this paper we extend the methodology for route travel time estimation and test its performance using simulation. The originality is defining cumulative plots for each exit turning movement utilising historical database which is self updated after each estimation. The performance is also compared with a method solely based on probe (Probe-only). The performance of the proposed methodology has been found insensitive to different route flow, with average accuracy of more than 94% given a probe per estimation interval which is more than 5% increment in accuracy with respect to Probe-only method.

1 Introduction

Travel time estimation is an important area of research and number of models with various degrees of complexities ranging from simple regression (Sisiopiku and Rouphail 1994; Sisiopiku, Rouphail et al. 1994), traffic flow theory (Nam and Drew 1999; Oh, Jayakrishnan et al. 2003), pattern recognition (You and Kim 2000; Bajwa, Chung et al. 2003; Robinson and Polak 2005; Coifman and Krishnamurthy 2007), to advance neural network (Park and Rilett 1998; Chen and Chien 2001; Liu, Van Zuylen et al. 2006) are proposed. Researchers have also applied data fusion techniques (Choi and Chung 2002; El Faouzi 2006) to fuse data from different sources, specifically detector and probe vehicles, with the aim to improve the accuracy and reliability of the estimates. Most of the researches on travel time estimation are limited to freeways However, travel time estimation on urban network is more challenging as mentioned below:

a) Interruptions in flow due to conflicting areas: On urban networks external control such as, traffic signals are needed to ensure safety at intersections. The flow on urban network not only depends on vehicle to vehicle friction but also on the external factors resulting in interrupted traffic flow. Vehicles are at stop-and-go running conditions and the delays experienced at the intersections are significant part of the travel time on an urban link. Hence, the spot-speed from a detector cannot be correlated to travel time on a link between intersections. In addition to the delays at the intersections, vehicles are also prone to mid-link delays due to a number of reasons such as, pedestrians, vehicles entering from side-streets, on-street bus stops
etc. There can be significant variation in travel time between two consecutive vehicles depending on the time when the vehicle arrives at an intersection. For instance, if the leading vehicle arrives during signal green phase and the following vehicle arrives during signal red phase then the following vehicle has to stop at intersection resulting in significantly higher travel time. Therefore, average travel time estimation solely based on probe data requires significantly large number of probes per estimation interval.

b) Significant proportion of flow to/from mid-link sinks/sources: The proportion of such flows is dynamic and varies with time of the day and day of the week. Generally, detectors are not installed on mid-link sinks/sources. Practically, the loss/gain of flow to/from a mid-link sink/source is unknown. Models solely based on detector data only capture the flow at the detector location and its performance can significantly deteriorate in the presence of significant flow to/from mid-link sinks/sources. Also, the performance is affected by the errors in detector counting.

c) Average link travel time may not be representative of travel time for different exit movements: An urban link is associated with different exit turning movements for instance, travel time for through, left and right exit movements. For ITS applications (such as route guidance) one is more interested in movement specific travel time than average link travel time. Movement specific travel time is more complicate to estimate than average travel time on the whole link.

The methodology developed in this research addresses the above issue by fusing the data from multiple sources (detector, signal, and probe vehicle). It is based on classical analytical procedure of estimating travel time using cumulative plots as explained in the following section.

2 Classical analytical procedure for travel time estimation

Cumulative plot is a graph of a function that defines the cumulative number of values (counts of vehicles passing over a detector) at time \( t \), starting from an arbitrary initial count, e.g., at \( t=0 \). Cumulative plots are used as a tool in number of engineering applications such as mass curve analysis in hydraulic engineering. In traffic engineering, Newell (1982) is a pioneer to use cumulative plots for dynamic analysis of deterministic congested systems.

Cumulative plot is monotonically increasing and can be assumed to be differential with respect to time. The slope of the plot at time \( t \) is \( CP'(t) \). The value of the cumulative counts at time \( t \) is \( CP(t) \). For time \( t \) and \( t+\Delta t \), the difference in the corresponding cumulative counts \( (CP(t+\Delta t) - CP(t)) \) gives the traffic counts during the time interval \( \Delta t \). The average flow during the time interval is the ratio of counts and time interval i.e., \( (CP(t+\Delta t) - CP(t))/\Delta t \).

Refer to Figure 1, two cumulative plots \( U(t) \) and \( D(t) \) are obtained at locations upstream \((u/s)\) and downstream \((d/s)\) of a road, respectively. Assuming: a) First-In-First-Out (FIFO) discipline is respected for all vehicles traversing from \( u/s \) to \( d/s \) (i.e., there is no vehicle overtaking); and b) the vehicles are conserved (i.e., there is no loss or gain of vehicles between \( u/s \) and \( d/s \)). The vertical distance (along \( Y \)-axis) between the two plots at time \( t \) defines the instantaneous number of vehicles \((n)\) between the two locations. The horizontal distance (along \( X \)-axis) for count \( i \) defines the travel time \((tt)\) for the \( i^{th} \) vehicle. The classical analytical principle for average travel time estimation defines the total travel time for all the \( N \) vehicles departing during the travel time estimation interval \((TE)\) from the location \( d/s \) as the area \((A)\) between the two cumulative plots. Average travel time per vehicle is the ratio: \( A/N \). Interested readers can refer to Page 1-24 of Newell (1982) and Chapter 2 of Daganzo (1997) for complementary reading. Even if FIFO discipline is not respected, the area \((A)\) between the two plots represents the total travel time as long as all the vehicles which arrive at
upstream during time \( t_1 \) and \( t_2 \) actually depart at downstream during time \( t_3 \) and \( t_4 \), and vice versa. Here \( t_1 \) and \( t_2 \) are time corresponding to the start and end of \( U(t) \) represented in the area, respectively; and similarly \( t_3 \) and \( t_4 \) are time corresponding to the start and end of \( D(t) \) represented in the area, respectively.

**Figure 1: Classical analytical procedure for average travel time estimation**

For the application of the aforementioned classical procedure, not only cumulative plots should be accurately estimated but also there should not be relative deviation amongst the plots (RD). The ideal situation is when detectors are perfect (i.e., they provide accurate vehicle by vehicle information) and vehicles are conserved between the upstream and downstream locations. However, these conditions are difficult to obtain in practice, especially in urban networks due to reasons mentioned below:

I. **Detector Error**: Loop detectors even under normal running conditions have counting error of around 5%. However, for cumulative plot these errors are also cumulative and can result in exponential relative deviation amongst the plots.

II. **Mid-link sources and sinks** such as, parking, mid-link street, residential and commercial areas etc., violate the requirement for conservation of vehicles between the two locations where cumulative plots are defined.

III. **Unknown cumulative plots for different link movements**: An urban link can have complex combinations of the flow to and from a link. For instance, shared lane at upstream link with unknown real turning proportions can complex the process of estimating cumulative plot at upstream location. Moreover, for exit movement specific travel time, the unknown cumulative plot for each exit movement is also to be estimated.

### 3 Cumulative plots and probe vehicle data integration

We have developed a methodology named CUmulative plots and PRobe Integration for Trafﬁc timE estimation (CUPRITE) by integrating cumulative plots and probe vehicle data to address the issues related to RD. The basic of CUPRITE is developed and tested in Bhaskar et al., (2009) for a link between two consecutive signalized intersections for only through movement at downstream intersection. In this paper, we extend the methodology for the estimation of route travel time with consideration of different exit movements at downstream.

For exit movement specific travel time estimation we need to know the cumulative plots at upstream \( U_m(t) \) and downstream \( D_m(t) \) for \( m^{th} \) movement. Assuming detectors at stop-line location: one can accurately obtain departure cumulative plot for each exit movements \( (D_m(t)) \). The stop-line detectors at upstream intersection provides total upstream cumulative
plot \((U_T(t))\) i.e., cumulative plot based on the total flow at the upstream entrance of the link. What is unknown is the upstream cumulative plot for each exit movement \((U_m(t))\).

For simplicity of discussion here we use the term exit moments. To be precise we consider the combination of different movements, based on the link geometry and signal phases. For instance: for downstream intersection, in Figure 2a, travel time for all the movements is to be differentiated. Here downstream cumulative plot for Rt, Lft and Thru movements are obtained from detector \(d_{a1}, d_{a2}\) and \(d_{a3}\), respectively. For downstream intersection, in Figure 2b, travel time for right movement is to be differentiated from a combination of through and left movements. Here, downstream cumulative plot for: Rt movement is obtained from detector \(d_{b1}\); and Thru+Lft movements is obtained from the sum of counts from \(d_{b2}\) and \(d_{b3}\).

**Figure 2: Urban link with different flow combinations**

The architecture for the CUPRITE to take into account the exit movement specific travel time estimation is provided as follows (see Figure 3):

**Step 1:** Integrate stop-line detector with signal timings to estimate \(U_T(t)\) and \(D_m(t)\);  

**Step 2:** Probe vehicle data is fixed to \(D_m(t)\) and points from where \(U_m(t)\) should pass are defined;  

**Step 3:** Initial estimate for \(U_m(t)\): Initial upstream cumulative plot for a movement is defined by vertically scaling the total upstream cumulative plots with scaling factor defined with historical database;  

**Step 4:** Redefine \(U_m(t)\) by vertical scaling and shifting technique;  

**Step 5:** Apply the classical procedure between redefined \(U_m(t)\) and \(D_m(t)\) to define the average travel time.

The details of the above steps are provided in the following sub-sections.

**Step 1: Integrating stop-line detector counts with signal timings**

Here, if the detector data is individual vehicle data (pulse data), then the cumulative counts can be obtained by cumulating the vehicles. However, if detector data is not a pulse data but an aggregated traffic count during certain detection interval (for instance counts per 60 seconds), then cumulating the counts for each detection interval will not reflect the actual traffic fluctuations within the detection interval. The granularity of the cumulative plots depends on the aggregation interval.

On signalised arterials, traffic is in stop-and-go running conditions i.e., vehicles have to stop at intersection during signal red phase, and the stopped vehicles form a queue and during
signal green phase the vehicles from the queue are discharged at saturation flow rate. These fluctuations can be captured by integrating the stop-line detector counts with signal timings, where the counts during the signal red phase are assigned to zero, and counts during the signal green phase are segregated into counts from the saturation flow and counts from non saturation flow. Refer to (Bhaskar, Chung et al. 2010) for the methodology integrating signal timings with aggregated traffic counts from detector data for accurate representation of cumulative plots.

**Figure 3: CUPRITE architecture for movement specific link travel time estimation**

**Step 2: Probe data fixed to D_m(t) and points from where U_m(t) should pass are defined**

**Real probe**

Here probe vehicle is defined as a vehicle which can provide time stamp when at an intersection (position where cumulative plots are generated). Generally probe vehicle is equipped with GPS. There are issues related with probe vehicle data such as, frequency of data, map-matching of data, urban cannon etc. Addressing such issues is beyond the scope of this paper. We assume, known value of time, \( t_u \) and \( t_d \) when probe vehicle is at upstream and downstream intersection, respectively.

**Relation between cumulative plots and probe**

Cumulative plot corresponds to the flow of vehicles at a specific point in space whereas; probe data corresponds to the probe vehicle only.

Figure 4a represents the cumulative plots, where, we define the time corresponding to the \( f^{th} \) rank in the plots as \( t_u' \) and \( t_d' \) for \( U(t) \) and \( D(t) \), respectively. Figure 4b represents the time space trajectory of a probe vehicle. If we fix the probe to the \( D(t) \), i.e., define \( t_d = t_d' \) and assign its rank as \( D(t_d) \) in the cumulative plots then, \( t_u' \) may not be equal to \( t_u \). This is due to RD amongst the cumulative plots in addition to non-FIFO traffic behaviour. Therefore, we define a parameter \( \Delta t \) (1) as follows:

\[
\Delta t = t_u' - t_u = U^{-1}(D(t_d)) - t_u
\]
Figure 4: Example for relationship between cumulative plots and probe

If there is no relative deviation amongst the cumulative plots then $\sum \Delta t$ for all the vehicles represented within the cumulative plots should be zero for both FIFO and non-FIFO system. Due to this property the area between the plots is total travel time, as long as the vehicles represented in the $U(t)$ and $D(t)$ are same.

Probe vehicles are a random sample from the population of vehicles. We make a hypothesis that if we fix probe vehicles to $D(t)$ and redefine $U(t)$ such that $\sum \Delta t$ for all the probes is zero then we should be able to estimate travel time accurately using $D(t)$ and redefined $U(t)$.

Virtual Probe

Virtual probe is defined as a virtual vehicle (not a real vehicle) that, during under-saturated traffic condition, departs from the downstream at the end of signal green phase and its travel time is free-flow travel time of the link ($t_{ff}$).

Figure 5: Illustration of a virtual probe

For under-saturated traffic conditions vehicle queue should vanish at the end of each signal effective green phase ($t_{GE}$) and travel time for the vehicle entering the intersection at time $t_{GE}$ should be close to $t_{ff}$. (Refer to Figure 5) Hence, if the following conditions for virtual probe are satisfied then we can define virtual probe such that it is observed at upstream and downstream at time $t_{GE} - t_{ff}$ and $t_{GE}$, respectively (i.e. $t_u = t_{GE} - t_{ff}$ and $t_d = t_{GE}$):

1. Absence of source for significant mid-link delay such as, on-street bus stop or mid-link intersections: As the travel time of a virtual probe is defined as free-flow travel
time of the link, therefore on the study link the sources for significant mid-link delay should be absent.

II. **Under saturated traffic condition with no-leftover-queue at the end of signal green phase.** Virtual probes are defined only for under-saturated conditions with logics of zero queue length at the end of signal green phase. For this the counts during the signal cycle should be less than the capacity as follows:

\[ D(t_{GE}) - D(t_{GE} - c) + \Delta < s^*g \]  \( (2) \)

Where: \( t_{GE} \), \( c \), \( s \) and \( g \) are end of the signal effective green time, signal cycle time, saturation flow rate and effective signal green time, respectively; \( s^*g \) is the capacity and \( \Delta \) is a calibration parameter to take into account the error in the estimation of capacity.

III. **Presence of RD i.e., the following equation should be satisfied:**

\[ U^{-1}(D(t_{GE})) - t_{GE} \notin [t_g - \delta, t_g + \delta] \]  \( (3) \)

Where \( \delta \) is a calibration parameter taking into account the variation in the estimation of \( t_f \). It can be considered equal to the standard deviation of the estimate of \( t_f \).

**Points from where \( U_{m}(t) \) should pass**

Say, we have \( n \) probe vehicles and the database for the probe is defined as list of \([t_u]\) and list of \([t_d]\) where the size of each list is \( n \). The value of \( f^{th} \) element in the list represents the data from the \( f^{th} \) probe.

The list \([t_u]\) and \([t_d]\) is appended with additional elements satisfying the conditions for virtual probe. If the conditions are satisfied, then for each under-saturated signal cycle: (a) time corresponding to the end of the green time \( (t_{GE}) \) is appended to the list \([t_d] \); and (b) \( (t_{GE} - t_f) \) is appended to the list \([t_u] \).

Following steps defines the points from where \( U_{m}(t) \) should pass

I. Sort the list \([t_d]\) in ascending order of its values. This is required as the rank of the probe in the cumulative plots is defined based on \( D_m(t) \).

II. Sort the list \([t_u]\) in ascending order of its values. This is required to make sure that the redefined \( U_{m}(t) \) is monotonically increasing and satisfies the property of \( \sum \Delta t = 0 \).

III. The required points through which \( U_{m}(t) \) should pass are \((t_{uj}, D(t_{dj}))\); where \( t_{uj} \) and \( t_{dj} \) are \( f^{th} \) value in the sorted list of \([t_u]\) and \([t_d]\), respectively.

**Step 3: \( U_{m}(t) \) from \( U_T(t) \)**

Let us consider an example. Figure 6 illustrates a study link with flow from three different directions at \( u/s \) and exit flow towards three different movements at \( d/s \). In the example, at \( u/s \): detector A and detector C are on shared-used lane with proportion of counts \( \eta_A \) and \( \eta_C \), respectively towards the study link. One can obtain total cumulative plot at \( u/s \) \( (U_T) \) of the study link as a linear combination of cumulative plots from each upstream detector, scaled with respect to the counts proportions:

\[ U_T = \eta_A CP_A + \eta_B CP_B + \eta_C CP_C \]  \( (4) \)

Where: \( \eta_A \), \( \eta_B \) and \( \eta_C \) are proportion of counts observed at upstream detectors A, B and C, respectively towards the study link. Here, \( \eta_B \) is unity as detector B is not on a shared-use lane.
To estimate the upstream cumulative plot for each movement we consider vertical scaling technique on $U_T$. Here we define scaling factors: $S_{Lft}$, $S_{Thru}$, and $S_{Rt}$ as the factors used to vertically scale $U_T$ to define upstream cumulative plot for each movement.

$$U_{Lft} = f(S_{Lft}, U_T); \quad U_{Thru} = f(S_{Thru}, U_T); \quad U_{Rt} = f(S_{Rt}, U_T) \quad (5)$$

Say variables $d$, $p$, and $m$, represent day of the week, time period of the day, and $m^{th}$ exit turning movement, respectively. The variable $S_{m,p,d}$ represents the scaling factor for the $m^{th}$ exit movement, the $p^{th}$ period of the $d^{th}$ day of the week. For instance: $S_{Lft,7:00,7:15am,Monday}$ is a scaling factor for left exit movement, from 7:00 a.m. to 7:15 a.m. on Monday. The cumulative plot for $m^{th}$ movement ($U_m(t)$) can be defined as follows:

$$U_m(t) = U_m(t_{s,p}) + S_{m,p,d}[U_T(t) - U_T(t_{s,p})] \quad \forall Time Periods and \forall t \in [t_{s,p}, t_{e,p}] \quad (6)$$

Where: $t_{s,p}$ and $t_{e,p}$ is the time corresponding to the start and end of the $p^{th}$ time period.

The procedure to estimate scaling factor is explained after the following step.

**Step 4: Redefine $U_{Rf}(t)$**

We define reference point as the point in which we have confidence that it is a correct point on the plot. $U_m(t)$ and $D_n(t)$ are initially two independent cumulative plots. When the traffic condition is free-flow (for instance during night) then counts for cumulative plots can be initialized to zero. This is the initial reference point ($P_0$). Say [$P_1$, $P_2$, $P_3$, ..., $P_n$] is the list of $n$ points from where $U_m(t)$ should pass then for redefining $U_m(t)$ for point $P_i$, the reference point is $P_{i-1}$.

Say, we have: a) a reference point ($t_{ref}$, $U_m(t_{ref})$); and b) point ($t_p$, $Y_p$) through which $U_m(t)$ should pass. Then, (Refer to equations (7), (8) and (9) and Figure 7) we redefine $U_m(t)$ by applying correction on it such that all points on the plot:

I. Before time $t_{ref}$ has no correction;
II. Between time $t_{ref}$ to time $t_p$ are scaled vertically; and
III. Beyond time $t_p$ are shifted vertically so that the redefined curve is continuous at time $t_p$ and is parallel to $U_m(t)$.
Arterial travel time estimation: Revisiting the classical procedure

\[ U_m(t) = U_m(t) + \text{Correction} \quad (7) \]

\[ \text{Correction} = \begin{cases} 0 & \forall t \leq t_{\text{Ref}} \\ (\text{scale} - 1) \ast (U_m(t) - U_m(t_{\text{Ref}})) & \forall t_{\text{Ref}} < t < t_p \\ (\text{scale} - 1) \ast (U_m(t_p) - U_m(t_{\text{Ref}})) & \forall t \geq t_p \end{cases} \quad (8) \]

\[ \text{scale} = \begin{cases} \frac{Y_p - U_m(t_{\text{Ref}})}{U_m(t_p) - U_m(t_{\text{Ref}})} & \text{if } U_m(t_p) \neq U_m(t_{\text{Ref}}) \\ 1 & \text{if } U_m(t_p) = U_m(t_{\text{Ref}}) \end{cases} \quad (9) \]

Figure 7: Redefining \( U_m(t) \) using vertical scaling and shifting technique.

How to define the scaling factor?
Here we consider two different approaches, to define the scaling factor, based on: historical average turning ratios; and historical effective scaling factor. Both are defined in terms of time of the day and day of the week.

Historical average turning ratios
As \( U_T \) is the total counts observed at the upstream entrance of the link, therefore the initial estimate for the scaling factor should be the actual real time exit turning ratio of the link.

Turning ratios are random variables and vary with time. It’s estimation is mathematically a non-deterministic problem and models (Martin 1997; Lan and Davis 1999) are developed to estimate the “most likely” solution. The models in literature can be applied for developing the historical database for average turning ratios values for different time of the day and day of the week and hence the scaling factor can be defined based on the most appropriate value.

Say variable \( \alpha_{m,p,d} \) represents the historical average turning ratio for \( m^{th} \) exit movement, during \( p^{th} \) time period and \( d^{th} \) day of the week. Then we can define scaling factor as:

\[ S_{m,p,d} = \alpha_{m,p,d} \quad (10) \]
**Historical effective scaling factor**

The testing result of the CUPRITE in the previous study (Bhaskar, Chung et al. 2009) gives confidence in the accurate estimation of travel time for application of CUPRITE with at least one probe per estimation period. Hence, the redefined cumulative plot at upstream with probe data can be utilized for developing a historical database of effective scaling factor for different time of the day and day of the week. The effective scaling factor incorporates the scaling required for exit turning ratio and also due to probable loss/gain of vehicles to/from mid-link sinks/sources.

To develop the database, at the end of each day, $U_T(t)$ and $U_m(t)$ are integrated to define the effective scaling factor for time periods with at least one probe vehicle. Say variable $s_{m,p,d}(11)$ represents the scale for a record of the $m^{th}$ exit movement, the $p^{th}$ time period of the $d^{th}$ day of the week:

$$s_{m,p,d} = \frac{Y_{T,d,p} - Y_{m,d,p}}{Y_{T,d,p}}$$

Where:

$$Y_{T,d,p} = U_T(t_e,p) - U_T(t_s,p)$$
$$Y_{m,d,p} = U_m(t_e,p) - U_m(t_s,p)$$

Where: $t_{e,p}$ and $t_{s,p}$ are the time corresponding to the start and end of the $p^{th}$ time period; $Y_{T,d,p}$ and $Y_{m,d,p}$ are the total counts observed, and counts for the $m^{th}$ movement observed during the $p^{th}$ time period, respectively (see Figure 8e).

The database consists of the values of the effective scaling factor $s_{m,p,d}$ properly classified in corresponding time of the day and day of the week. The database is daily self updated, with the new values defined at the end of the day. The required scaling factor $S_{m,p,d}$ can be defined as the median of values of effective scaling factor defined in the historical database:

$$S_{m,p,d} = \text{Median of } s_{m,p,d}$$

For better understanding an example is illustrated in Figure 8. Say we have a historical database of the scaling factors, either defined in terms of turning ratios or effective scaling factor. For each period, first the scaling factor from historical database is obtained and initial estimate of the upstream cumulative plot for movement $m$ i.e., $U_m(t)$ is defined using equation (6). Thereafter, $U_m(t)$ is redefined by integrating the cumulative plots with probe vehicle data. Finally, the redefined $U_m(t)$ is utilized to self update the historical database using equation (11).
4 Route travel time

For route travel time estimation we consider the following two approaches:

4.1 Component Based (R_C)

Here we divide the entire route into different components and route travel time is the sum of time-slice travel time from each component. The component is a link between two consecutive signalised intersections. We consider the pairs of cumulative plot at upstream and downstream for each component. Each pair of cumulative plot is independent from the other pair in the network and RD amongst each pair is corrected independently. Here: \( U_{c,m}(t) \) and \( D_{c,m}(t) \) represent a pair for \( m \)th movement of component (link) \( c \); If we have \( n \) components, then \( c = 1,2,3,..,n-1, n \). Where \( n \) is the downstream most component and 1 is the upstream most component. We are interested in estimating average travel time for vehicles that depart the route during time \( t_{s,n} \) to \( t_{e,n} \).

For this, we first look at the downstream component (component \( n \)) and define average travel time during time \( t_{s,n} \) to \( t_{e,n} \). Then, we look at the time from \( t_{s,n-1} \) to \( t_{e,n-1} \) during which the vehicles are observed at the upstream component (component \( n-1 \)) where: \( t_{s,n-1} = U^{-1}_{n,m}(D_{n,m}(t_{s,n})) \) and \( t_{e,n-1} = U^{-1}_{n,m}(D_{n,m}(t_{e,n})) \) and define average travel time for all vehicles that depart during \( t_{s,n-1} \) to \( t_{e,n-1} \) from component \( n-1 \). This process is repeated for further upstream components and so on. The sum of the travel time for each component is the route travel time (Figure 9a)

4.2 Extreme based (R_E)

Here we estimate route travel time by directly considering the area between the cumulative plots at extreme points of the route i.e., upstream entrance and downstream exit of the route.

For better understanding a self explaining example for RC and RE approaches is illustrated in Figure 9 where we are interested in estimating route travel time from point S to point E. There are three different components SA, AB and BC. Figure 9a and Figure 9b illustrate the procedure for RC and RE, respectively.

Figure 9: Example for RC and RE

### 5 Testing under controlled environment

The methodology is tested using simulated data from AIMSUN. A network of five consecutive signalized intersections, with stop-line detectors is considered (see Figure 10) and we define a route from intersection A to intersection E. Different AIMSUN API’s are written to extract signal timings, detector counts and individual vehicle data. Probe vehicles are randomly selected from individual vehicle data. For each travel time estimation period: a) Actual average travel time (actual) for the route is obtained from the simulated vehicles that traverse the complete route; b) CUPRITE provides the estimated average travel time (estimated). The performance of the CUPRITE is evaluated in terms of accuracy (13), where first for each travel time estimation period absolute percentage deviation (15) is obtained thereafter, mean absolute percentage error is defined (14).

\[
A_M(\%) = (1 - MAPE) \times 100
\]

\[
MAPE = \frac{\sum_{i=1}^{N} Error_i}{N}
\]

\[
Error_i = \frac{|actual_i - estimated_i|}{actual_i}
\]
Here, first $R_C$ and $R_E$ estimation techniques are compared for flow $F_1$ (where 90% of the flow at upstream traverses the route). Thereafter, the result of $R_E$ application is provided for following flow values:

- **F2**: 50% of the flow at upstream traverses the route
- **F3**: 20% of the flow at upstream traverses the route

Flow $F_1$ is analogous to a route with major traffic flow. Flow $F_2$ and $F_3$ are analogous to route where there is significant traffic in-flow and out-flow within the route. Two different case studies are performed:

**Case M1**: Here the comparison between $R_C$ and $R_E$ approaches is performed for flow combinations $F_1$ and for: under-saturated (Case M1.U); and over-saturated traffic condition (Case M1.O).

**Case M2**: Here different flow combinations ($F_1$, $F_2$ and $F_3$) are analysed for $R_E$ approach and compared with a method solely based on probes “Probe-Only” method.

Within a travel time estimation interval there can be certain number of probes. “Probe-Only” method defines average travel time of probes. Its comparison with $R_E$ evaluates if there is significant benefit of integrating cumulative plots with probes.

For $R_C$ the components defined are through movements from $A$ to $B$; $B$ to $C$; $C$ to $D$; and $D$ to $E$. For $R_E$ cumulative plots at upstream entrance, at intersection $A$, and downstream exit at, intersection $E$, are considered.

### 5.1 Case M1

Figure 11 is the graph of accuracy versus fixed number of probes per estimated period ($S_n$) for under-saturated (case M1.U) and over-saturated (case M1.O) traffic conditions. During under-saturated traffic condition, virtual probe can be defined for each component and hence even in the absence of real probe accurate travel time can be obtained for $R_C$ ($A_M > 96\%$ for $S_n = 0$) (see Figure 11a). During over-saturated traffic condition, virtual probes are not considered (because conditions for virtual probes are not satisfied) and the accuracy of $R_C$ increases with increase in $S_n$ (see Figure 11b).

Accuracy of $R_C$ is slightly higher than that from $R_E$. Though $R_C$ is more accurate but detectors data and signal timings are required for each component. There are higher chances of getting probe for each component than one traversing the complete path. $R_E$ is simple to apply and data only at upstream and downstream of the route is required but the required probe should traverse the complete route, which could be less frequent.
5.2 Case M2

In the previous section it is demonstrated that $R_{C}$ has better performance than $R_{E}$. Therefore, in this section we perform further testing using $R_{E}$. This provides lower bound for the performance as the approach $R_{C}$ can slightly improve the accuracy. The results for the three different flows $F1$, $F2$ and $F3$ are presented in Figure 12.

With at least one probe per estimation interval the performance of CUPRITE is generally more than 95% and increases with increase in number of probes. Whereas, significantly large number of probe vehicles are required to obtain comparable accuracy from Probe-only method.

With less number of probes there is significant benefit of integrating cumulative plots with probe vehicle. For instance for $S_n = 1$ there is more than 5% improvement in accuracy. The availability of large number of probes per estimation period is quite rare and it demonstrates the significant benefit of integrating cumulative plots with probes.

For the above analysis the “true” average travel time for the route is obtained by all the vehicles that traverse the complete route. For $F3$ (see Figure 12c) only 20% of the vehicles traverse the complete route. Therefore, for large $S_n (>15)$ the accuracy from Probe-only method is significantly higher.

The above analysis indicates that CUPRITE can be applied for route travel time estimation for different flow combination with implicit consideration of mid-route delay due to presence of mid-route intersections.
6 Conclusions

One of the major limitations of the existing travel time estimation models is that it estimates average travel time for the whole link. Generally to estimate movement specific link travel time, penalties are added to the average link travel time. For ITS applications more robust and accurate movement specific travel time is required. This paper utilizes a methodology based on classical analytical procedure for travel time estimation. It extends the model based on integration of cumulative plots with probe for movement specific travel time and route travel time estimation. Two different approaches Component based and Extreme based, are discussed for route travel time estimation. Both the approaches provide similar results. Component based is more reliable with greater chances of probe vehicle in each interval, though additional data from each component is required. Extreme based is simple, and only requires data from upstream and downstream of the route but chances of obtaining a probe that traverses the entire route might be low.
7 References


