The Price of Travel Time for Household Activities: A Theoretical Insight

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Abstract

In Australia, the proportion of time allocated to travel activities has increased steadily since the early 1990s. Various studies have developed estimates of travel time for different users and travel conditions. The value assigned to time is a key factor in the discrepancy between the disparate conclusions. The value of travel time includes waiting as well as actual travel. The general consent is that people have fixed travel time budgets. However, the review of the literature reveals that there is no simple answer on how to measure travel time. This paper is a stepping stone towards achieving this end. I assume that the main determinant of the demand for transport infrastructure is time; hence, the demand for travel time reflects the demand for transport infrastructure. Further, I disaggregate the demand by types of activities. Assuming only two types of activities (market and household) I investigate the demand for travel time for household activities. I then focus on highly-skilled women, and use high-skills as a proxy for high-income. By developing a variation of the generalized Cobb-Douglas utility function, I argue that changes in the distribution of education (income) reflect the distribution of time inequalities in travel time for household activities. The distribution of time inequality affects the price elasticity of demand for travel time (and for transport infrastructure) and the equilibrium price. This investigation has new implications for transport policies (but not only).

Keywords: travel time, recreation, price-elasticity, income-elasticity
1. Introduction

In Australia, the proportion of time allocated to travel activities has increased steadily since the early 1990s. Various studies have developed estimates of travel time for different users and travel conditions. The value of travel time, for example, is one of the main elements in determining investments decisions in transport infrastructure, and the main element of congestion costs. It accounts for 40-50% of total costs in the decision to invest in transport infrastructure, and 70-80% of total congestion costs. Hence, total costs are heavily influenced by the value assigned to travel time. However, while there is general agreement that there are gains from the consistent application of agreed values, there is no simple answer on how to measure travel time (BITRE, 2009).

The value of travel time includes waiting as well as actual travel. It includes the costs to consumers of personal (unpaid) time spent travelling and the costs to businesses of paid employee time. The general consent is that people have fixed travel time budgets (BITRE, 2009). As a result, speed, reliability, and savings of travel time become important.

The value assigned to time is a key factor in the discrepancy between the disparate conclusions; however, the review of the literature reveals that there is no simple answer on how to measure travel time. In this paper I consider the value of travel time for consumers of personal (unpaid) time spent travelling. It is a stepping stone towards achieving a uniform measure of travel time.

In this paper I assume that time is the main determinant of the demand for transport infrastructure, rather than distance, number of trips and use of land, as currently assumed. Hence, time is the final constraint on the demand for travel time. Further, I consider travel time as a necessity, rather than a disutility (as currently assumed) and, hence, travel time should be considered an input as well as an output of utility. I argue further that travel time for household activities is a complement to these activities. I disaggregate the demand for travel time by types of activities. I assume that there are only two types: i) market and ii) household activities. I investigate the demand for travel time for household activities, and I focus on high-skills women. High-skills are a proxy for high-income. Further, I measure the price of travel time in terms of quantity rather than money. For example, if people spend seven hours a week in travelling for household activities, then seven hours out of the total hours in one week (about 4% of
168 hours) reflects also the ‘price’ value of the budget of travel time out of total income, which needs to be constant.

I then develop a variation of the generalized Cobb-Douglas utility function to argue that changes in the distribution of education (income) reflect the distribution of time inequalities in travel time for household activities. The distribution of time inequality affects the price elasticity of demand for travel time (and for transport infrastructure), and the equilibrium price. This investigation has implications for transport policies (but not only). This paper is divided in the following sections. Section 2 presents some trends in women’s travel time; section 3 defines the demand of travel time, section 4 is a discussion on preferences for travel time, section 5 investigates the income elasticity of demand for travel time, and section 6 is my conclusion.

2. Estimates of Travel Time for Women

In Australia, from 1991 to 2006, travel time estimates of women in Melbourne show a rise in the proportion of time allocated to travel time (Figure 1, average weekly hours).

Figure 1: Travel time, women, Melbourne, 1991-2006

Source: Ironmonger (2008)
Since 1991, travel time for household activities increased by 2.02 hours. In particular, by 2006, travel time for consumption activities grew by 87% (Table 1).

Table 1: Growth in hours spent travelling, women, in Melbourne 1991-2006

<table>
<thead>
<tr>
<th>Women Average hours per week, by purpose</th>
<th>1991</th>
<th>2006</th>
<th>Difference</th>
<th>Percentage Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment</td>
<td>0.47</td>
<td>0.50</td>
<td>+ 0.3</td>
<td>+ 7%</td>
</tr>
<tr>
<td>Consumption</td>
<td>1.54</td>
<td>2.88</td>
<td>+1.34</td>
<td>+ 87%</td>
</tr>
<tr>
<td>Household Work</td>
<td>3.23</td>
<td>3.31</td>
<td>+ 0.08</td>
<td>+ 5%</td>
</tr>
<tr>
<td>Market Work</td>
<td>1.37</td>
<td>1.94</td>
<td>+ 0.57</td>
<td>+ 6%</td>
</tr>
<tr>
<td>TOTAL</td>
<td>6.61</td>
<td>8.63</td>
<td>+ 2.02</td>
<td>+30.56%</td>
</tr>
</tbody>
</table>

Source: Ironmonger (2008)

The data also reveal that women’s preferred mode of travelling is car driving. Their total hours of travel time by car rose from 5.1 hours (average, per week) in 2006 to 6.1 hours (average, per week) in 2008 (Ironmonger, 2008). The theory of consumer demand assumes that individuals allocate time according to changes in prices (to minimise the costs of travelling and maximise the benefits from saving time in travelling), maintaining utility of income constant.

The Cobb Douglas utility function always assumes a positive amount of income (Varian, 1996), or, in this instance, of time (and money) for travel time. However, the demand of the good for household activities, and hence, for travel time, is positive only for some values, and in this paper I argue that the demand is positive only for some values of time and income. For example, when income is zero, the demand of time for travelling is zero; hence, the demand of travel time for household activities is only relevant when a positive amount of time and market goods is consumed.

In this paper, I consider the utility of income in terms of a time-budget rather than only a money-budget of time for travelling, and I define this time-budget as a necessity; consequently, money- and time- budgets for household travelling are a necessity. The variable to consider when measuring the budget of travel time is recreation.

I define recreation ‘r’ as \( r = T - w - u_w \), where \( T \) is total time, \( w \) is paid work time, and \( u_w \) is unpaid work time (which includes travel time for household activities) (Cavagnoli, 2008). Only if we value (or measure) unpaid work time (including travel time for household activities) in the same way as paid-work time, then \( r = T - w \), or \( r = 24 - w \)
(as assumed by the theory); otherwise, time for recreation becomes a fixed ratio of total time (or total income), and travel time is a fixed ratio of total time for household activities. Travel time for household activities is a complement to these activities, and recreation is a complement to work activities, and hence, a fixed cost (Cavagnoli, 2008).

Within this definition, if time is allocated according to changes in the relative prices of market and non-market activities, then, for utility to remain constant, the ratio of time (and income) to recreation activities, r, must remain constant, as well as the travel time budget for household activities. This means that if, for highly-skilled (high-income) people, women in particular, time for travelling increases, then time for work (paid or unpaid) must decrease, so that the proportion of time for recreation activities remain constant (i.e. leisure is a normal good). The evidence in Australia however, is quite the opposite. In addition to the increasing trend in travel time for household activities, the average time for paid and unpaid work, is increasing (Duxbury and Higgins, 2008; Vanrooy, Oxenbridge, Buschanan, Jakubauscas, 2007); consequently, the time allocated to recreation activities has decreased. This means that utility of income is not constant, and that leisure is not a normal good.

The evidence suggests that since the early 1990s, the value of the initial endowment \((\omega)\) of time (and income) for household activities (including travel time and recreation) is decreasing with respect to the value of time in paid work. This change would explain why some economists argue that that savings in travel time are a myth, but also that for highly-skilled (high-income) people, time for leisure has become a necessity.

If we re-define recreation activities as a necessity, and travel time for these activities a complement (hence, a necessity), and value these goods in quantity terms (as quantity ratios of total time), then it becomes clear that there is a fixed budget of time (and income) allocated to travel time for household and recreation activities. Any savings of time, and hence any mode (or type of road) chosen to save time, becomes a luxury good. If, for example, we observe that is a complete switch to driving mode (section 2), and if driving is the most preferred mode to save time, then the next question to ask is about whether this switch is indeed efficient for high income earners.

In other words, does car driving (travelling for household activities) save time and money? Are the long term benefits (returns) from investing in efficient cars, constant, or are they decreasing at a faster than expected rate? Are the costs of an extra unit of time in travelling lower or higher than the average cost of running a car in the long term?
The answer to these questions depends on the price or value of the endowment of time allocated to household activities ($\omega$), and the time in travelling required, with respect to the value of the endowment time for recreation activities (non work), and for paid work. If the proportion of time for recreation remains constant (utility constant with respect to the output of time for paid and unpaid work), then the long term benefits from investing in efficient cars would be greater than the initial and ongoing costs of buying and maintaining a car, and of consuming time for travelling (road services). Women high-income earners would then demand (and consume) more time and income to save time in travelling (i.e. luxury good) relative to low-income earners, and this choice would be efficient.

However, if the time allocated to travelling for household activities increases in proportion to the time allocated to market work, while time for recreation decreases by the same proportion, and if the costs of driving (maintenance, time and money) increase, then, the quantity (price) of time in travelling becomes more expensive to maintain and to consume, than what was expected initially (at the initial value of the endowment). Hence, while for high-income earners, the time and money budget for household travelling increases; savings of time in travelling decrease, which means that savings become a necessity rather than a luxury. That is, I expect a low price elasticity of demand for (fast, comfortable and fuel efficient cars, and fast roads), while for low-income earners, any fast mode of travelling becomes too expensive to buy.

If the price of time for household and recreation activities (value of the initial budget) decreases relative to the price of time for market activities (price of road services and modes of travelling), then the costs of travelling for household activities increase at an increasing rate relative to the benefits from consuming market goods and services (i.e. buying time with cars and fast roads).

In this paper I argue that either there is not enough supply of transport infrastructure (i.e. services from roads), to meet the increased demand, and-or there exists an unacknowledged externality which increases the costs and decreases the value of time in household activities (and travelling) relative to the value of time in paid work. Given that recreation is not recognised as a necessity as yet, and given that time for household travelling is not recognised as a complement for those activities as yet (budget for travelling for household activities as a necessity), it seems reasonable to argue that the supply of infrastructure is limited given the unacknowledged externality (lack of information). This means that the extra (above the average budget) time that
women allocate to travelling on roads i) increases the demand for transport infrastructure; ii) increases the demand for fast modes of travelling (i.e. cars and toll roads); but also that this shift in demand increases the quantity of time relative to its price; iii) the ‘real’ cost of travel time for household activities, increases by more than expected; iv) the value of the market price of time, if constant, v) leads to an increase in the costs of purchasing the budget of travel time, while vi) decreasing the value of the time budget for household activities.

The Cobb-Douglas utility function cannot account for these extra costs; and the price elasticity of demand given by the ‘generalised’ cost approach, cannot account for the influence of these costs on the elasticity of demand. This is the reason why I propose and alternative function in section 3.

3. The Demand of Travel Time for Household Activities

In this paper I assume that the demand for travel time reflects the demand for transport infrastructure. I distinguish two types of activities and I consider the demand for travel time for household activities. Further, in order to investigate the net demand for household travelling, I assume that travel time is a complement to household activities and that: i) the ratio of the endowment of labour time to actual labour supply is constant; that ii) there exists a budget of time (and money) for household travelling and that is constant; iv) that recreation time is constant ratio of total time (and total income), and that v) the supply of time services from the transport infrastructure (a public good), is constant.

If we assume that X is the demand for commodity commuting ‘capital intensive’ with a greater share of market goods relative to travel time (of total income and total time), then Y is the demand for the commodity commuting ‘time intensive’, with a greater share of household time, relative to market goods (of total income and total time). If commuting is fast, is more capital intensive (expensive fuel efficient cars and/or ‘toll’ roads), hence, it is a luxury. The budget of time is more valuable relative to the price of capital and, hence, relative to the market price of time. If commuting is not fast, then is more time intensive (no choice of toll roads; but more public transports; no petrol efficient cars). The budget of time is less valuable relative to the price of capital and relative to the market price of time (value of the income).
If we assume that an individual may first consider how much of a necessary good to consume, such as time for commuting, then there exists a minimum time and money budget requirement to purchase the commodity travelling for household activities. If no time and income is left after buying time for the commodity travelling, individuals do not consume the ‘luxury’ commodity to save time in travelling for household activities (i.e. toll roads or fast and fuel efficient cars).

In other words, individuals whose value of time for travelling is greater than the minimum value required to buy time for these activities, will spend more of their income to save time in travelling; hence, they will consume more of fast roads, and fast and efficient cars, relative to other modes of transports, and relative to other consumers. Thus, the value of the budget share to travel time for household activities is dependent on the individual income, and not on the aggregate average income and average time for travelling.

The demand for travel time reflects the demand for transport infrastructure, but also the demand for modes of transport (i.e. cars, bikes, busses, trains, roads) and time-saving (more expensive) modes (i.e. fast, fuel-efficient and comfortable cars, fast lanes). If there is income and/or time dispersions, dispersion affects the equilibrium price of travel time and of transport infrastructure. As I consider high-skills as a proxy for high-income, the market price of travel time for household activities will rise with i) high skills, ii) per capita income, but also, with iii) per capita time for household activities (including recreation, and household travelling), and iv) a decrease in per capita supply of transport infrastructure (time services from roads and other modes).

I consider utility as a function of a minimum quantity of time consumed in travelling for household activities and a minimum amount of income to buy travel time. The model reflects an individual (i) utility function of the following type:

\[ U_i = (X_i + C)\alpha Y_i^\beta, \quad i = 1, \ldots, n \quad \alpha, \beta > 0 \]

where \( X_i \) is the individual i’s demand of time for household activities (including travelling), and, \( Y_i \) is the demand for time as a composite good (i.e. commuting = time + market goods). C is a shift parameter and is positive. Without the shift parameter C, the utility function would reflect the Cobb-Douglas utility function. The shift parameter C can either represent i) the constant ratio of total time to travel time (time-budget for household activities) at the recreation-endowment point \( \alpha \) (given the fixed supply of transport
infrastructure); or ii) C can represent a constant cost-share of net earnings. Either way, substitution between time (and income) for household, recreation and travel activities, and consumption, is limited.

If we assume that the allocation of time (and income) responds to changes in prices (i.e. wage rate), utility is then unambiguously held constant at that level of time for household, recreation and travel activities. Therefore, when we consider the demand of travel time in terms of household (and recreation) and market activities, the demand for this good is a gross demand, and the net demand (for consumers) is positive only for some values of income (Varian, 1996). That is, only when the value of the endowment of recreation time is held constant (or does not decrease); there exists a minimum budget of time (and money) for travelling.

If we assume separable and quasi-linear preferences, as in the Cobb-Douglas utility function, and if we accept that there is substitution between time and income for household and market activities, then preferences take this special form only when an individual's income (at the endowment of recreation time, and given the fixed supply of transport infrastructure) is greater than some threshold level, \( w^0 \). Then for consumers, the income (not the price) \( w^0 \) becomes a function of the ratio of budget shares, the market price of goods (i.e. efficient modes), the market price of time, and the shift parameter C.

This is the reason why I consider an alternative function to the Cobb-Douglas for preferences that are assumed to be separable, but they are so only in the presence of a fixed endowment (budget of time and money), otherwise, preferences cannot be separable. Hence, a minimal level of travel time (and income) has to be consumed, irrespective of its market price and the consumer's income. To note that the sum of all the proportions of the goods consumed must equal 1. This alternative approach can generate an income elasticity of demand for time in travelling that is greater than one, but the price elasticity of demand for time in travelling falls with increases in education and hence, per capita income, and per capita time (given the fixed supply of transport infrastructure). The next section considers consumers' preferences.
4. Preferences

In this paper, utility for the individual $i$ is represented by:

$$U_i = (X_i + C)\alpha Y_i^\beta,$$

$i = 1, \ldots, n$  \hspace{1cm}  $\alpha, \beta > 0$

(1)

Where $X_i$ is individual $i$’s demand of time for household activities (including travelling) and, $Y_i$ is the demand of market time for commuting (a composite good). $C$ is a shift parameter and is positive, which makes preferences non-homothetic. It is identical for all individuals. Hence, the budget shares are identical for all individuals. Consumption of $X$ and $Y$ are constant with respect to increases in income. Without the shift parameter $C$, utility is the usual Cobb-Douglas function with homothetic preferences and unit elastic demand for travel time with respect to income, total time and price. This utility function reflects quasi-homothetic preferences. The wealth expansion path does not start at the origin, but it is linear.

The budget constraint for individual $i$ is:

$$w_i = p_x X_i + p_y Y_i$$

(2)

where $p_x$ and $p_y$ are prices of $X$ and $Y$, respectively; $w_i$ is individual $i$’s income level at the fixed endowment of labour time to actual labour supply (at a fixed share of total time to household and market activities, given $w^0$ at $\alpha$, at the initial endowment of hours $\omega$). The marginal rate of substitution between $X$ and $Y$ is:

$$MRS_{xy} = [dU_x/dX_i] / [dU_y/dY_i] =$$

$$= - [ \alpha (X_i + C)^{\alpha-1} Y_i^\beta ] / [ \beta (X_i + C)^\alpha Y_i^{\beta-1} ] = - \alpha/\beta Y_i / (X_i + C)$$

(3)

Note that when $C$ is zero, the marginal rate of substitution is identical to that of the traditional Cobb-Douglas problem:

$$MRS_{xy}^{C-D} = [dU_x/dX_i] / [dU_y/dY_i] = - \alpha/\beta Y_i / X_i$$

(4)

In this model, the indifference curves are everywhere flatter relative to those of the Cobb-Douglas model (Figure 2). In maximizing utility, an individual must consume the amounts of $X$ and $Y$ where the indifference curve is tangent to the budget line. This point
occurs when income (and time for household travelling) is greater than the minimum income requirement \( w_i > w^0 \) (demand for travel time N).

**Figure 2: Indifference curves with homothetic (dotted) and non-homothetic preferences**

The income-expansion path is shifted upwards (along Y-axis). To solve the utility maximization problem, the first order conditions from the Lagrangian are the following:

\[
L_i = (X_i + C)^\alpha Y_i^\beta + \lambda_i (w_i - p_x X_i - p_y Y_i)
\]

\[
dL_i/dX_i = \alpha (X_i + C)^{\alpha-1} Y_i^\beta - \lambda_i p_x \leq 0, X_i \geq 0
\]

\[
dL_i/dX_i = \beta (X_i + C)^{\beta-1} Y_i^\beta - \lambda_i p_y \leq 0, Y_i \geq 0
\]  \hspace{1cm} (5)

\[
dL_i/d\lambda_i = w_i - p_x X_i - p_y Y_i = 0
\]

Equation (5) yields the following specification of the demand of time for travelling (or household activities, including travelling):

\[
X_i = \left[ \frac{\alpha}{\alpha+\beta} \right] [(w_i - w^0)/p_x], w_i > w^0
\]

\[
0, w_i \leq w^0
\]  \hspace{1cm} (6)
Where \( w^0 = (\beta C/\alpha) p_x \). The function of the demand for time as a market commodities is:
\[
Y_i = \frac{1}{(\alpha + \beta)} \left[ (\beta w_i + \alpha w^0) / p_y \right]
\]  
(7)

\( Y_i = w_i / p_y, \quad w^0 = w^0 \)

Equation (6) tells us that demand of time for household travelling is positive only if the value of an individual’s time (and income) is greater than some threshold level, \( w^0 \). The value of time given the minimum amount of income (not the price) \( w^0 \) is a function of the ratio of budget shares, the price of time for household activities, and the shift parameter. Since these are identical for all consumers, the threshold level of income (and its share of time in recreation) is the same for everyone. The Engel curve is illustrated in Figure 3.

**Figure 3: The Engel curve**

\[
X_i(w)
\]

This shows the demand of time for household travelling as a function of income, holding prices constant. It is kinked at the threshold level. The demand for travel time for \( Y \) also changes, depending on the individual’s income. When \( w_i = w^0 \), the individual allocates all of his/her income (and time) to consumption of \( Y \). In analysing the consumption of \( X \) and \( Y \), the ratio \( X/Y \) illustrates the relationship between the present model and the usual Cobb-Douglas utility specification. The ratio is:

\[
X_i / Y_i = \frac{\alpha (w_i - w^0) / (\alpha + \beta) p_x}{[(\alpha + \beta) p_y / \beta w_i + \alpha w^0]} = \frac{p_x}{p_y} \left[ ((\alpha w_i - \alpha w^0) / (\beta w_i + \alpha w^0)) \right] \]  
(8)
As an individual’s income increases, this consumption ratio also increases:

\[
\frac{d(X_i/Y_i)}{dw_i} = \frac{(p_y/p_x)\left[(\alpha \beta w_i + \alpha w^0) - \beta (\alpha w_i - \alpha w^0)\right]}{\left(\beta w_i - \alpha w^0\right)^2}
\]

\[
= \frac{(p_y/p_x)\left[\alpha (\alpha + \beta)w^0\right]}{\left(\beta w_i - \alpha w^0\right)^2} > 0
\]

\[
d^2(X_i/Y_i)/dw_i^2 = -2(p_y/p_x)\frac{\alpha\beta(\alpha+\beta)w^0}{(\beta w_i - \alpha w^0)^3} < 0
\]

The first partial derivative in equation (9) says that by increasing income (and education), an individual’s consumption of X rises faster than the consumption of Y. In the limit, the ratio approaches the traditional Cobb-Douglas consumption ratio:

\[
\lim_{w_i \to w^0} X_i/Y_i = 0
\]

\[
\lim_{w_i \to \infty} X_i/Y_i = \frac{\alpha}{\beta} \frac{p_y}{p_x}
\]

This relationship is illustrated in figure 4. The reason the consumption ratio approaches the Cobb-Douglas ratio is that as the individual income rises, the effect of the minimum income (and time) requirements diminishes. If everyone’s income (and its value) increases systematically, eventually everyone’s income will be greater than the minimum requirement. As mentioned, the Cobb-Douglas specification does not require any minimum income and time requirements for consumption of both goods. In this context, the Cobb-Douglas function may be considered the asymptotic utility specification with respect to income.
5. The Income Elasticity of Demand for Travel Time (for $X > 0$)

The Cobb-Douglas specification results in unit income elasticities. With the shift parameter $C$, the income elasticities of demand for travel time are not unity. The income elasticity of demand for travel time in household activities is:

$$\eta_x = (dX/dw_i) (w_i/X_i) = \left[\frac{\alpha}{\alpha + \beta}\right] \left[\frac{1}{p_x}\right] \left[\frac{((\alpha + \beta) p_x)}{(\alpha (w_i - w^0))}\right]$$

(11)

If I consider travel time for household activities as a luxury good, the effect of increasing income on the income elasticity is negative:

$$d\eta_x/dw_i = (w_i - w^0 - w_i)/(w_i - w^0)$$

$$= - \left[\frac{w^0}{(w_i - w^0)}\right]^2 < 0$$

$$d^2\eta_x/dw_i^2 = [2w^0/(w_i - w^0)^4] > 0$$

(12)

As income rises, the travel time budget for household activities, becomes less of a luxury at an increasing rate, but will always remain a luxury good. This is illustrated in Figure 5.
The income elasticity of demand for travel time is:

\[ \eta_x = \frac{\partial Y}{\partial w} \]

The income elasticity of \( Y \) is:

\[ \frac{dY_i}{dw_i} = \left[ \frac{\beta}{\alpha+\beta} \right] \left( \frac{1}{p_y} \right) \]

\[ \eta_y = \left( \frac{dY_i}{dw_i} \right) \left( \frac{w_i}{Y_i} \right) = \left[ \frac{\beta}{\alpha+\beta} \right] \left( \frac{1}{p_y} \right) \left( \frac{w_i}{Y_i} \right) = \left[ \frac{\beta}{\alpha+\beta} \right] \left( \frac{1}{p_y} \right) \left( \frac{w_i}{Y_i} \right) \]

\[ = \left[ \frac{\beta w_i + \alpha w_0}{\beta w_i + \alpha w_0} \right] < 1 \quad (13) \]

The effect of increasing income on the income elasticity of \( Y \) is positive:

\[ d\eta_y/dw_i = \left[ \frac{\beta (\beta w_i + \alpha w^0) - \beta^2 w_i}{(\beta w_i + \alpha w^0)^2} \right] > 0 \]

\[ d^2\eta_y/dw_i^2 = \left[ -2\alpha\beta^2 w^0 / (\beta w_i + \alpha w^0)^3 \right] < 0 \quad (14) \]

\( Y \) is a necessary good. As income rises, \( Y \) becomes less of a necessary good at a decreasing rate, but will always remain a necessary good. Figure 5 shows that the
income elasticity of Y with respect to income is a positive, concave function, with an upper limit of one. That is, Y is considered a necessary good by all individuals regardless of income level.

In the usual Cobb-Douglas specification, the price elasticity of demand for travel time is unity. Here, with non-homothetic preferences, the price elasticity of demand for travel time for \( X > 0 \). The demand of travel time for household activities is elastic. It is this feature of demand for travel time that drives the results. Individual demand of time for household activities:

\[
X_i = \alpha/(\alpha+\beta) \left[ (w_i - w_0)/p_x \right] = \frac{1}{\alpha+\beta} \left[ (\alpha w_i - \alpha w_0)/p_x \right] 
\]

\[
= \frac{1}{\alpha+\beta} \left[ (\alpha w_i/p_x) - \beta C \right]
\]

The price elasticity of demand for travel time \( X \) (time for household activities, including travel time) is:

\[
\varepsilon_x = \left( \frac{dX}{dp_x} \right) \left( \frac{p_x}{X_i} \right) = - \left[ (\alpha/(\alpha+\beta)) \left( w/p_x^2 \right) \left( \frac{(\alpha+\beta)p_x}{(\alpha w_i - \beta p_x C)} \right) \right]
\]

\[
= - \left[ \alpha w_i / (\alpha w_i - \alpha w_0) \right] = - \left[ w_i / (w_i - w_0) \right]
\]

\[1 \varepsilon_x \] \( = w_i / (w_i - w_0) \quad > 1\)

\[
\lim_{w_i \to w^0} 1 \varepsilon_x = w^0 / (w^0 - w^0) = + \infty
\]

\[
\lim_{w_i \to \infty} 1 \varepsilon_x = \infty / (\infty - w^0) = 1
\]

The limits in equation (16) tell us that demand for travel time will remain elastic at every income (and education) level. As an individual's income rises, his demand for travel time will become relatively less elastic at an increasing rate:

\[
d\varepsilon_x / dw_i = (w_i - w^0 - w_i)/(w_i - w^0)^2 = - \left[ w^0 / (w_i - w^0)^2 \right] < 0
\]

\[
d^2 \varepsilon_x / dw_i^2 = 2 w_i w^0 / (w_i - w^0)^4 > 0
\]
Figure 6 illustrates this relationship. As income tends to infinity, the income elasticity of demand for household travel time approaches one (the Cobb-Douglas price elasticity). That is, the demand of travel time for household activities is elastic for all individuals, regardless of income level. As income rises, the price elasticity of demand for travel time in absolute value decreases. The demand for travel time of high-income (high-skills) individuals is price inelastic relative to the demand for travel time of low-income (low-skills) individuals.

**Figure 6: Price elasticity of demand for household travel time**

![Graph showing the price elasticity of demand for household travel time.

The demand of time for $Y$ is: $Y_i = \left(\frac{1}{\alpha + \beta}\right) \left[\frac{\beta w_i + \alpha w^0}{p_y}\right]

\frac{dY_i}{dp_y} = - \left(\frac{1}{\alpha + \beta}\right) \left(\frac{\beta w_i + \alpha w^0}{p_y}\right) \frac{1}{p_y^2} \quad (18)

The price elasticity of demand for time for other goods $Y$ (time for market activities, including commuting) is: $\varepsilon_y = \left(\frac{dY_i}{dp_y}\right) \left(\frac{p_y}{Y_i}\right) =

= - \left[\left(\frac{1}{\alpha + \beta}\right) \left(\frac{\beta w_i + \alpha w^0}{p_y}\right) \frac{1}{p_y^2}\right] \left(\frac{\beta w_i + \alpha w^0}{\beta w_i + \alpha w^0}\right) = -1 \quad (19)

Demand of time for $Y$ is unit-elastic; that is, the elasticity does not vary with income level. That is $Y$ is a necessary good. If the minimum income requirement is a function of price (as a quantity share of total time), individuals with different education (per capita income) will also have different minimum requirements. The demand for travel time fluctuates
with education, per capita income, and per capita time for recreation and household activities.

Rising time inequality lowers the price elasticity of market demand for travel time (in absolute value) because high-income earners are assumed to get richer in terms of money and time. As a result, the equilibrium price rises. With increasing time inequality, in the aggregate, fewer people are able to meet, simultaneously, both the minimum time and money requirements.

Firstly, it is because an increase in inequality has the direct effect of lowering income levels for some people. Specifically, for those who met the minimum income requirement marginally, a decrease in income to less than the requirement will mean that they can no longer afford to consume time in recreation and market time saving goods. Secondly, an increase in inequality increases the equilibrium price of time for household activities relative to market activities, which also increases the minimum income and time requirements. This is an indirect effect that exacerbates the problem just described. With fewer people able to meet the minimum requirements, aggregate demand of time for household activities will decrease.

Future research should focus on how time inequality, per capita income (education) and per capita time for household activities affect the equilibrium price and demand of time for household travel time. An increase in dispersion increases the equilibrium price of time in recreation (as the ratios of time for paid work and travelling increase) at a decreasing rate (i.e. the relationship between equilibrium price and dispersion should be positive and concave). Dispersion decreases aggregate demand of time for X and increases aggregate demand of income for Y.

This means that holding i) the (quantity of) services of time from transport infrastructure constant, and ii) the quantity of recreation constant, then the price of travel time in the market equals the value (price) of travel time for household activities ($w^0 = w_i$).

If the average hours of travel time for household activities increases while the time allocated to household activities (and recreation), decreases, then the value of the initial budget of travel time decreases relative to the market value of time ($w^0 < w_i$). As $w^0$ represents also the value of travel time in the market, at the given supply of transport infrastructure (services), then the value of travel time for household activities is also decreasing relative to the market value of time. Hence, the value of time from consuming capital intensive commodities decreases relative to the market value of goods. The market value of time intensive commodities decreases with respect to the market value
of time. The commodity X becomes more (than expected) capital expensive, as the value of time decreases relative to the price of market goods; and Y becomes more (than expected) time expensive, relative to the market price of time.

As high-skills consumers allocate more time to work, relative to household work, than low-skills people, their income is high, but their quantity budget of time for travelling is lower relative to their income. However, as time-saving goods become more expensive, their demand for those (time) becomes price inelastic (time for travelling becomes a necessity). Low-skills individuals, allocate more time to household work, relative to market work, but the value of their budget decreases relative to the market value of time, so that their price elasticity of demand for time is also price inelastic (the money-budget for travelling becomes a necessity). Both cases display a negative income elasticity of demand for commuting.

While we generally believed that the excess demand for travel time, and hence, for transport infrastructure, is the result of a sub-optimal equilibrium market price, and hence, that either the price should be higher, or that taxes should be introduce to reallocate the costs, in this paper I argue that this ‘excess’ demand is the result of a decreased value of the consumers’ travel time budget relative to the market value of the budget, as we do not include travel time and recreation time as a necessity. This lack of acknowledgment leads to a lack of value (price) in cost-benefit analysis, which causes an underestimation of the costs (relative to the benefits) for consumers.

6. Conclusion

In this paper I develop a model of demand for travel time in household activities. Specifically, I develop a variation of the generalised Cobb-Douglas utility function. I focus on high- and low- skilled women to investigate how a change in the ratio of labour endowment to actual labour supply affects the demand of time for commuting. Given the general increase in the average hours of work in the market, I argue that changes in the distribution of education lead to inequalities in the distribution of time (and income) for household activities, including travel time.

The inequality of the distribution affects the price elasticity of demand for travel time, and therefore equilibrium price. As inequality (time) rises, the market demand for travel time becomes less price elastic. If for high-income earners the demand of travel
time for household activities is price-inelastic, time for paid work will not decrease for every increase in the wage rate. The results from this analysis are, conducive to nonlinear regression techniques so that time (and income) inequality may be tested as a source of variation of prices in travel time for household activities, across cohorts and, hopefully, across countries. To note also that in this paper I distinguished between the gross- and the net-demand for travel time. That is, in this paper I considered the net demand of consumers, given by high- and low-skilled consumers. However, the gross demand would also include producers (or sellers) of time. Future research will extend this analysis to focus on the effect of ‘sellers’ on the price of travel time.
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