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Modelling diversion plans during extensive disruptive events

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Abstract

When a natural disaster such as a flood or bushfire occurs, traffic incident management becomes a necessity. Traffic incident management (TIM) involves the systematic use of resources including human, technological and physical to lessen the impact of incidents. One of the key components of traffic incident management is the management of traffic in the outer cordon after the event. Diversion planning is a traffic management task that is challenging due to traffic dynamics and a large number of trip origins and destinations involved. During the long-term recovery phase of disaster management when some roads are closed for rehabilitation and reconstruction, diversion planning becomes even more vital to the road network.

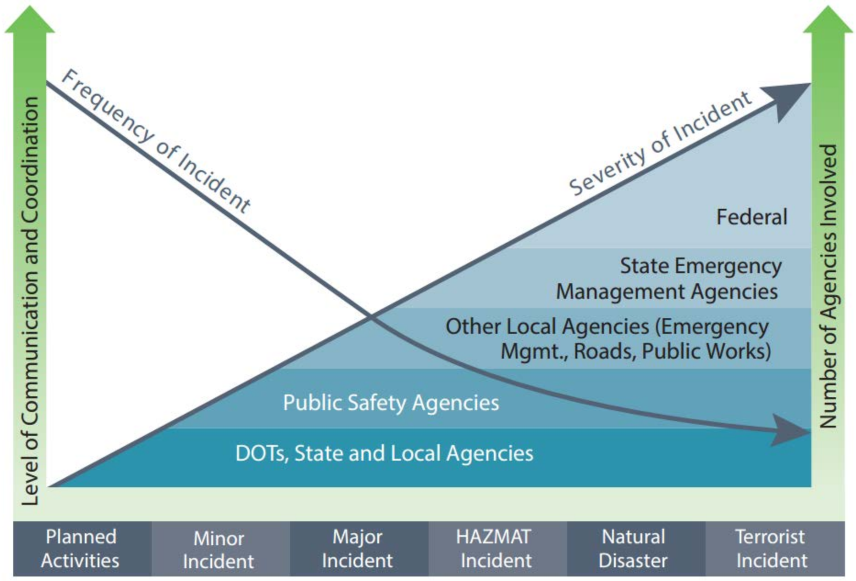
Regardless of what mechanism be used for traffic, models are required to identify the most relevant links on the remaining part of the network from which traffic needs to be diverted. The aim of such a model is to optimise the traffic performance throughout the network using guidance facilities such as Variable Message Signs (VMS). Moreover, finding the optimal paths to be recommended to road users is a key task to be undertaken as part of diversion planning. These optimal paths eventually aim to guide traffic in a way that transitions from pre-event equilibrium to post-event equilibrium faster and more smoothly. In so doing, it is argued in this paper that traffic should be guided to divert from shortest paths of post-event equilibrium condition. Obviously, it is unlikely to be able to guide traffic from all origins to all destinations in a large-scale disruptive event due to lack of physical resources. Therefore, this paper aims to answer the question of what is the best location to provide information using VMS to maximise the benefit to road users in the aftermath of large-scale disruptive events such as natural disasters.

1. Introduction

Diversion planning is highlighted as a complex component of traffic incident management (Carson 2010). Traffic Incident Management (TIM) involves the systematic use of resources including human, technological and physical to lessen the impact of an incident in time and severity dimensions and increase the efficiency of the system (Charles 2007). Depending on the scale and type of the incident, various organisations could be involved in traffic incident management such as road traffic organisations, police, and emergency service organisations. As far as the maintenance and operation of road and transport systems are concerned, road and transport agencies, as well as police, would take the lead during extreme events such as natural disasters. Recently, Climate Change Adaptation Guide for Transportation Systems Management, Operations, and Maintenance has highlighted how the number of agencies involved in an incident management increases with the severity of the incident. As shown in Figure 1, for natural disasters and terrorist incidents, state and federal emergency management agencies get involved in addition to the transport related organisations. Hence, traffic management for significant human-made or natural incidents would be actually part of disaster management. Moreover, traffic management includes a variety of actions and procedures to deal with the incident and safety issues.

An action that should be scrutinised as part of disaster management is traffic management around affected areas. One necessary action to manage traffic around the incident area is to provide drivers with adequate information about the incident and available diversion routes. That is why effective diversion planning could be considered as one of the key steps in disaster management that could be regarded as a potential measure of effectiveness (MOE). Similarly, traffic management during a disaster has recently been recognised as a recovery task in the, “Guide to Emergency Response Planning at State Transportation Agencies” (Wallace et al. 2010) and, “A Guide to Regional Transportation Planning for Disasters, Emergencies, and Significant Events” (Matherly et al. 2014). This implies that an appropriate recovery plan should also have also plans for managing traffic in a way that residual capacity of the network as a system is exploited optimally while the recovery efforts are in progress to eliminate the damage.

Figure 1: Involvement of agencies in incident management (Asam et al. , 2015)



Currently, road and emergency service authorities usually close all the damaged roads while the rehabilitation efforts are in progress. They also provide road users with some guidance in the surrounding of the affected areas. This approach obviates safety concerns but fails to take the road network performance after a disaster into account. Hence, a model is required to identify the most relevant links on the remaining part of the network from which traffic needs to be diverted from. The aim of such a model is to optimise the traffic performance throughout the network using guidance facilities such as Variable Message Signs (VMS). Not only is identifying these links important but also finding the optimal paths to be recommended to road users is a key task to be undertaken as part of diversion planning. These optimal paths aim to guide traffic in a way that transitions from pre-event equilibrium to post-event equilibrium faster and more smoothly. In so doing, we argue that road users should be provided with diversion guidance to ensure that the network reaches its post-disaster equilibrium condition faster. In essence, the guidance provided for the diversion leads road users to take shortest paths to their destinations as if they have perfect knowledge about the post-disaster functional and topological shape of the network. However, it is unlikely to be able to guide traffic from all origins to all destinations in a large-scale disruptive event due to constraints in physical resources. Therefore, this paper aims to answer the question of what are the best locations to provide information using VMS (or any other mechanisms) to maximise the benefit to road users in the aftermath of large-scale disruptive events such as natural or human-made disasters.

2. Road network performance under disaster

Before a disaster, the road network is in an equilibrium condition. Wardrop (1952) describes user equilibrium state as a situation in which no driver can reduce his travel time by choosing a new route (Patriksson 1994). It is based on the first Wardrop's principle which is concerned with route choice behaviour of greedy drivers who intend to decrease their travel time individually. The mathematical formulation of optimum UE leads to a nonlinear programming problem which is defined as:

$$\mathbf{min\,T(f)} = \sum_{\mathbf{a\epsilon A}} \int_{\mathbf{0}}^{\mathbf{fa}} \mathbf{t_a(s)ds} \tag{1}$$

s.t.

$$\sum_{\mathbf{r \in R_{pq}}} \mathbf{v_{pqr}} = \mathbf{d_{pq}} \qquad \forall (\mathbf{p}, \mathbf{q}) \in \mathbf{C}, \tag{2}$$

$$\mathbf{v_{pqr}} \geq \mathbf{0}; \quad \mathbf{f_a} \geq \mathbf{0}; \qquad \forall (\mathbf{p}, \mathbf{q}) \in \mathbf{C}, \qquad \forall \mathbf{r} \in \mathbf{R_{pq}} \tag{3}$$

$$\sum_{\mathbf{(p,q) \in C}} \sum_{\mathbf{r \in R_{pq}}} \boldsymbol{\delta}_{\mathbf{pqra}} \mathbf{v_{pqr}} = \mathbf{f_a} \qquad \forall \mathbf{a} \in \mathbf{A} \tag{4}$$

Where $\mathrm{R_{pq}}$ contains a set of potential paths available for origin-destination pair $(\mathrm{p}, \mathrm{q}) \in \mathrm{C}$. $\mathrm{v_{pqr}}$ is the flow (number of vehicles) on the route while $\mathrm{d_{pq}}$ is the demand. δ_{pqra} is the link-route incidence matrix in which one sits if route r uses link a and 0 otherwise. $\mathrm{f_a}$ is also total link flow. Finally, $\mathrm{t_a}(s)$ is calculated based on a link performance (congestion) function which also describes the dependency between traffic flows and travel times, or in other words, reflect the effect of congestion. The Bureau of Public Roads (BPR) developed a link congestion (or performance) function which has been used many different studies. This link performance (congestion) function defines average travel time ($\mathrm{t_a}(s)$) for a vehicle on a link as (5).

$$\mathbf{t_a(s)} = \mathbf{t_a}\left(\mathbf{1} + \mathbf{0.15}\left(\frac{\mathbf{f_a}}{\mathbf{c_a^{pre}}}\right)^{\mathbf{4}}\right) \tag{5}$$

Where t_a is free-flow travel time on link a per unit of time, f_a is flow (or volume) of traffic on the link and $\boldsymbol{c_a^{pre}}$ is the capacity of the link during pre-disaster period.

After a disaster, the operation of road network could be disrupted entirely or partially. A disruption is a dysfunction of an infrastructure or part of it that usually arises when part of its ingredients be it physical or a set of activities critical to the function of the system cannot operate as expected (Lee et al. 2007). Any disruptions across a road network take place as a result of damage to road elements or because of potential danger in using some parts of the system. Thus, the topological and functional characteristics of the road network change after a disaster. In other words, post-disaster capacity of some links ($\mathrm{c_a^{post}}$) would decrease due to damage. Therefore, let the post-disaster capacity of the links be defined as:

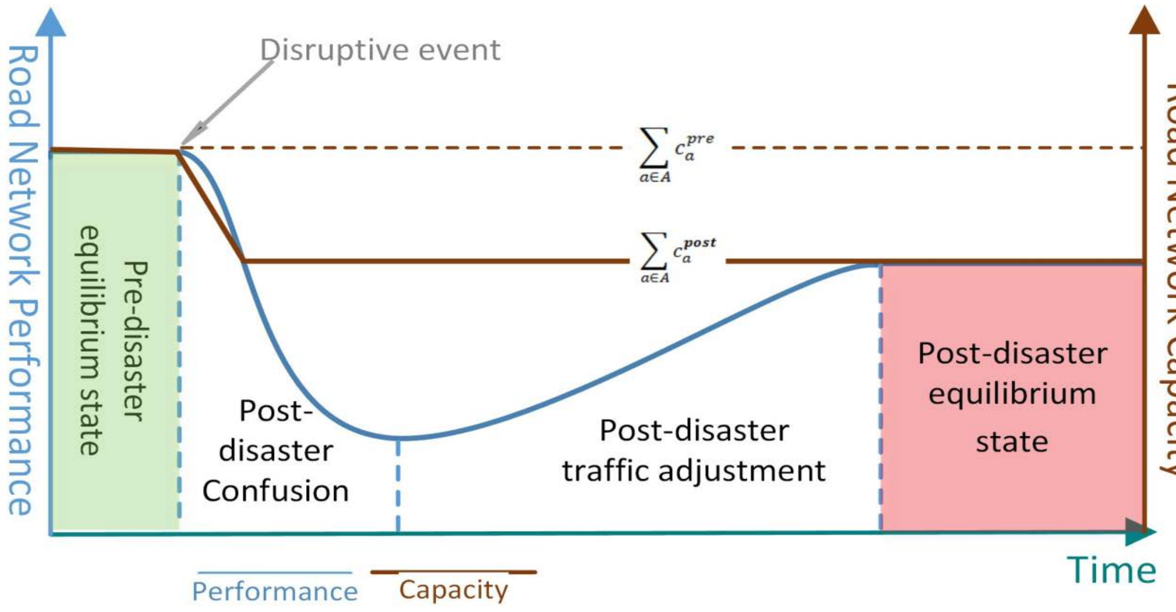
$$\mathrm{c_a^{post}} = \mathrm{c_a^{pre}} - \mathrm{R_a} \qquad \forall \mathrm{a} \in \mathrm{A} \tag{6}$$

s.t.

$$\mathrm{R_a} \in \{0,1,2,\ldots,\mathrm{c_a}\}.\ \forall \mathrm{a} \in \mathrm{A} \tag{7}$$

During the aftermath of a road network disruption, drivers' road choice behaviour is adjusted by the new capacity paradigm of the network. This adjustment process continues till traffic again reaches to a new equilibrium condition. Let these two equilibrium states be called pre- and post-disaster equilibrium states. This process is shown in Figure 2. Before a disaster occurs, road network and users are in an equilibrium state. Obviously, it means that total journey time is minimum in this state. In this pre-disaster equilibrium state, road network performance such as total travel time is at maximum level considering Wardrop's first principle. Then, total road network capacity reduces abruptly due to the destruction of some of the elements of the road network. It is assumed that some of the damage requires long-term recovery. In this situation, the pre-disaster equilibrium state collapses due to the reduction in the road network capacity. Immediately following a disaster, the loss of road network performance could be exacerbated since the traffic has not yet adjusted to the new conditions. This happens since the assumption of perfect knowledge about the road network is not valid anymore immediately after a disruptive event during the post-disaster confusion period (Faturechi & Miller-Hooks 2014). During the post-disaster traffic adjustment period, road network performance reaches its highest level again that matches the available residual capacity of the network after the disaster ($\sum_{a\in A} c_a^{post}$). This is called post-disaster equilibrium state established after some time following a disaster. Although it is not covered in Figure 2, this diagram could also continue until the road network elements are reconstructed. Each time the capacity of the road network increases due to the recovery of a damaged road network element, it takes some time for the traffic to adjust to the new capacity of the road network. Concurrently, the demand on the post-disaster road network returns to its pre-disaster level during the post-disaster traffic adjustment period as well. Thus, the assumption of demand inelasticity in long-term after a disaster is a reasonable even though road recovery is still in progress (Faturechi & Miller-Hooks 2014).

Figure 2: Pre- and post-disaster equilibrium states under disaster



3. Guided transition from one state to the other state

In a large-scale natural disaster, several geographically separated links could be disrupted. Thus, the current practice where guidance is provided at each location separately could fail to deal with interdependencies across the network. Lack of consideration for these interdependencies could lead to compromising social welfare. Casari and Wilkie (2005) argue that social welfare can be substantially affected due to the inefficiency of decisions during the recovery phase of disaster management. One of the root causes of such inefficiency is decentralised decisions after large-scale disasters. Hence, to make the most out of the residual capacity of a road network, it is vital to have a network-wide guidance plan for transitioning from pre-disaster to post-disaster equilibrium conditions.

Considering the two equilibrium states highlighted in the previous section and the need to consider guidance for the whole network, we suggest that traffic could transition from a pre-disaster equilibrium state to its post-event counterpart through a guidance mechanism within the network. This guidance could help to reach post-disaster equilibrium faster and thereby have a smooth transition after the disaster. In other words, such guidance would shorten the time needed for post-disaster traffic adjustment. This would be beneficial for the performance of the road network following a disaster. This more rapid transition based on guidance could be done via mechanisms such as Variable Message Sign (VMS). Regardless of what mechanism is chosen, the planner should deal with physical and human resource constraints. For applying VMS, the number of available facilities could be limited especially for a large-scale disaster. Therefore, it is necessary to find out the most critical links that would aid this transition across the network.

We argue those links that lose a higher proportion of their volume during post-event disaster equilibrium are more important to be provided with the guidance. For instance, assume that link A loses 50% of its traffic flow during post-disaster user equilibrium, It should be considered more important for guiding traffic in comparison with link B that loses just 10% of its traffic. With governing the traffic at the entrance to link A to go through other paths to their destinations, the post-disaster equilibrium state could be reached earlier if either of link A or B should be chosen to receive this guidance. Therefore, to find the M-most critical links to provide guidance for, the problem can be formulated as:

[CLS: Critical Links Search]

$$\min \sum_{a \in A} (f_a^{Pre} - f_a^{Post}) u_a \tag{8}$$

s.t.

$$\sum_{a \in A} u_a \leq M \tag{9}$$

$$u_a = \begin{cases} 1 & \text{if there exists a path with a disrupted link } (b \in A^D) \\ & \text{that comes after a in the path;} \\ 0, & \text{Otherwise.} \end{cases} \quad , \forall a \in A \tag{10}$$

$$u_a \in \{0,1\} \tag{11}$$

$$M \in \mathbb{N} \tag{12}$$

u_a is equal to 1 if link $a \in A$ receives guidance and is otherwise zero. $f_a^{Pre}$and f_a^{Post} are known flows on link $a \in A$ for pre-disaster and post-disaster equilibrium state respectively. These two variables are estimated through a UE traffic assignment having post-disaster network capacity (c_a^{post}). M is an integer that is given by the network manager to determine the number of available VMS facilities that can be deployed. Equation (10) ensures that only links on which flow will be faced with a disrupted link are chosen. The reason for requiring this condition is to avoid selecting links that do not involve disrupted flows. A^D includes the set of disrupted links. To solve this problem, a brute-force search is applied using the following steps:

- Step 1: Calculate $f_a^{diff} = f_{\boldsymbol{a}}^{\boldsymbol{Pre}} - f_{\boldsymbol{a}}^{\boldsymbol{Post}}$ for each link.
- Step 2: Sort all links based on f_a^{diff}.
- Step 3: Choose the top M links to be provided with guidance.

To provide road users with the proper guidance at the entrance to the identified critical links, the post-disaster traffic assignment explained previously is exploited. In essence, the routes to the destinations in the OD matrix that traverse a critical link in post-disaster equilibrium state are used to determine the detours. In doing so, the assignment proportion matrix is considered. Let $\mathrm{P} = [\ldots, \mathrm{p}_{\mathrm{pqa}}, \ldots]$ be the proportion matrix. This represents the proportion of traffic on link $a \in A$ that is concerned with the demand on the OD pair $(\mathrm{p}, \mathrm{q}) \in \mathrm{C}$ (Cho 2001). Having P, the optimum paths at the entrance of each critical link identified (achieved by [CLS]) to all destinations leads to a network-wide post-disaster equilibrium being produced. These paths should be communicated to the road users by which network can reach to post-disaster equilibrium state as early as possible.

4. Illustrative examples and discussion

A network-based representation of a road network is shown in Figure 3 to illustrate how this method. We assume that some links within the network are disrupted due to a network-wide disaster. Then, using the proposed method, the M most critical links that need guidance are found. This network includes 110 directed arcs and 45 nodes. It consists of 15 nodes in grey that are origins and destinations which sum up to 150 pairs of Origins/Destinations (OD). A method similar to the one described in Kaviani et al. (2015) is used to estimate the demand on the network using traffic count data. This demand estimation method was originally proposed by (Spiess 1990). Table 1 presents the known traffic volumes on links in the network. Having these known volumes, the demand on the network from each origin to each destination is estimated that results in a total demand of 20533 vph on the network. This demand leads to 20402 hours as total travel time across the network in the pre-disaster equilibrium state. Note that links with the shortest distance in the network have a free flow travel time of 0.1 hour. Free flow travel times on links that are twice as long as the shortest links (eg. 28→38) are 0.2 hours while those that are three times the length of the shortest links (eg. 37→38) are 0.3 hours.***

In the following sections, two scenarios are presented to be solved by the proposed method. In each scenario, a set of links is assumed to be disrupted and then, the ten most critical links that should be provided with guidance are shown. In scenario A, links that are disrupted are longer and distributed more widely across the network compared with scenario B where the disrupted links are more geographically-concentrated.

Figure 3: Network for assessment of the method

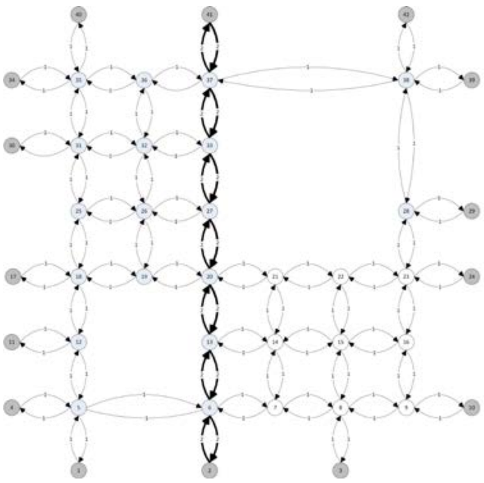


Table 1: Known traffic counts on the network

Origin	Destination	Count (vph)
16	23	2000
18	19	2000
5	12	2000
8	7	2000
19	18	2000
11	12	1000
27	33	4000
16	23	2000
32	33	1500
34	35	500
38	39	500
42	38	1500
40	35	1000
9	8	1500
27	20	3500
23	22	1500
31	32	1500

4.1. Scenario A

In this scenario, links 36↔37, 13↔20, 16↔15 and 26↔25 are assumed to be damaged and need long-term reconstruction. In this situation, the total travel time of the network becomes 24236 hours once the post-disaster equilibrium is reached. This means an increase of above 4000 hours in total travel time within the network. This figure could rise much further due to drivers' lack of knowledge about the incident in the immediate moments after the incident. If we assume that all affected road users will reroute when they get to the disrupted links on their route and go the new shortest route, the total travel time of network becomes 30125 hours (the bottom of the curve of road network performance in Figure 2). However, reaching to this maximum utilisation of the road network after a disaster could take time. The later this post-disaster equilibrium and minimum total travel time is reached, the more travel costs are imposed on the road users and economy. Our proposed method shows that links 27→20, 33→27, 6→13, 14→13, 31 →35, 32→36, 26→32, 37 →33, 15→14 and 7→14 are the ones that respectively should be provided with guidance facilities since the traffic volume on them drops the most during post-disaster equilibrium. Figure 4 illustrates where these links are located on the network. Also, Table 2 shows links chosen as well as the value for f_a^{diff} for each of them. Based on our argument, the links with higher f_a^{diff} in this table should be considered to have the guidance variable message signs. Thus, if four VMS facilities are available for example, the first four links have the highest priority to be equipped with VMS. Similarly, for n number of available facilities, the top n links should be chosen to get VMS installed on them.

Figure 4: Critical links to be provided with guidance in scenario A

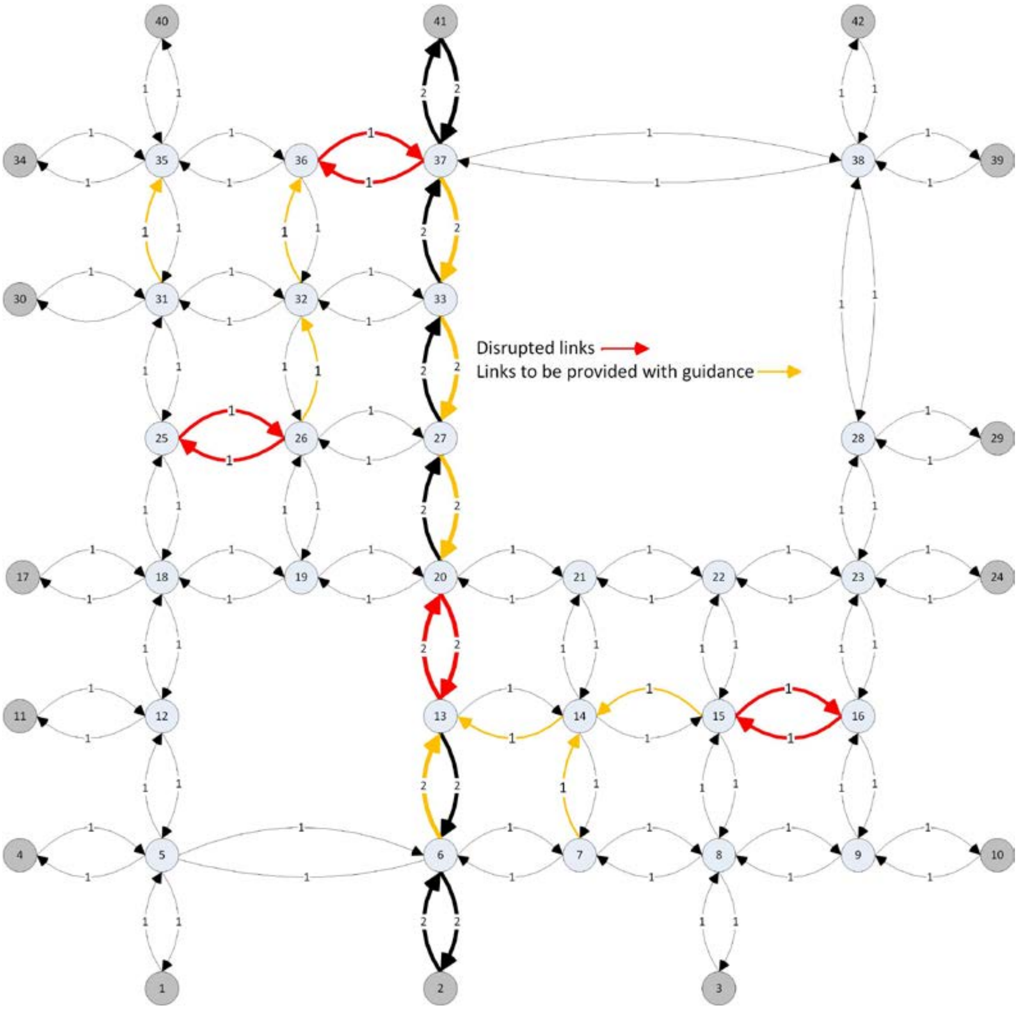


Table 2: The identified critical links on which users should be provided with guidance

From	To	$f_a^{diff} = f_a^{Pre} - f_a^{Post}$
27	20	1566
33	27	1279
6	13	1279
14	13	961
31	35	957
32	36	890
26	32	617
37	33	560
15	14	299
7	14	274

Knowing that link 27→20 is a critical link for the transition to post-disaster equilibrium, Table 3 illustrates what should be communicated to the road users at the intersection before cars enter link 27→20.

Table 3: Guidance and diversions needed to be communicated on the node 27 (intersection) for scenario A

Source	Destination	Path
27	17	27 -> 26;26 -> 19;19 -> 18;18 -> 17;
	11	27 -> 26;26 -> 19;19 -> 18;18 -> 12;12 -> 11;
	4	27 -> 26;26 -> 19;19 -> 18;18 -> 12;12 -> 5;5 -> 4;
	1	27 -> 26;26 -> 19;19 -> 18;18 -> 12;12 -> 5;5 -> 1;

It is interesting to note that vehicles on link 31→35 are provided with guidance and this link is a considerable distance away from disrupted areas. The reason for is that with link 36→37 disrupted, new post-disaster shortest paths to destinations 42, 41 and 39 do not traverse link 31→35 according to the proposed model (Table 4). Thus, road users need to divert to another path if their destination is one of these links. The diversions and guidance would lead to a faster transition to post-disaster equilibrium condition in which road network total performance is maximised by the fact that in equilibrium condition no one can decrease their travel time anymore.

Table 4: Guidance and diversions needed to be communicated on the node 31 (intersection) for scenario A

Source	Destination	Path
31	42	31 -> 32;32 -> 33;33 -> 37;37 -> 38;38 -> 42;
	41	31 -> 32;32 -> 33;33 -> 37;37 -> 41;
	39	31 -> 32;32 -> 33;33 -> 37;37 -> 38;38 -> 39;

4.2. Scenario B

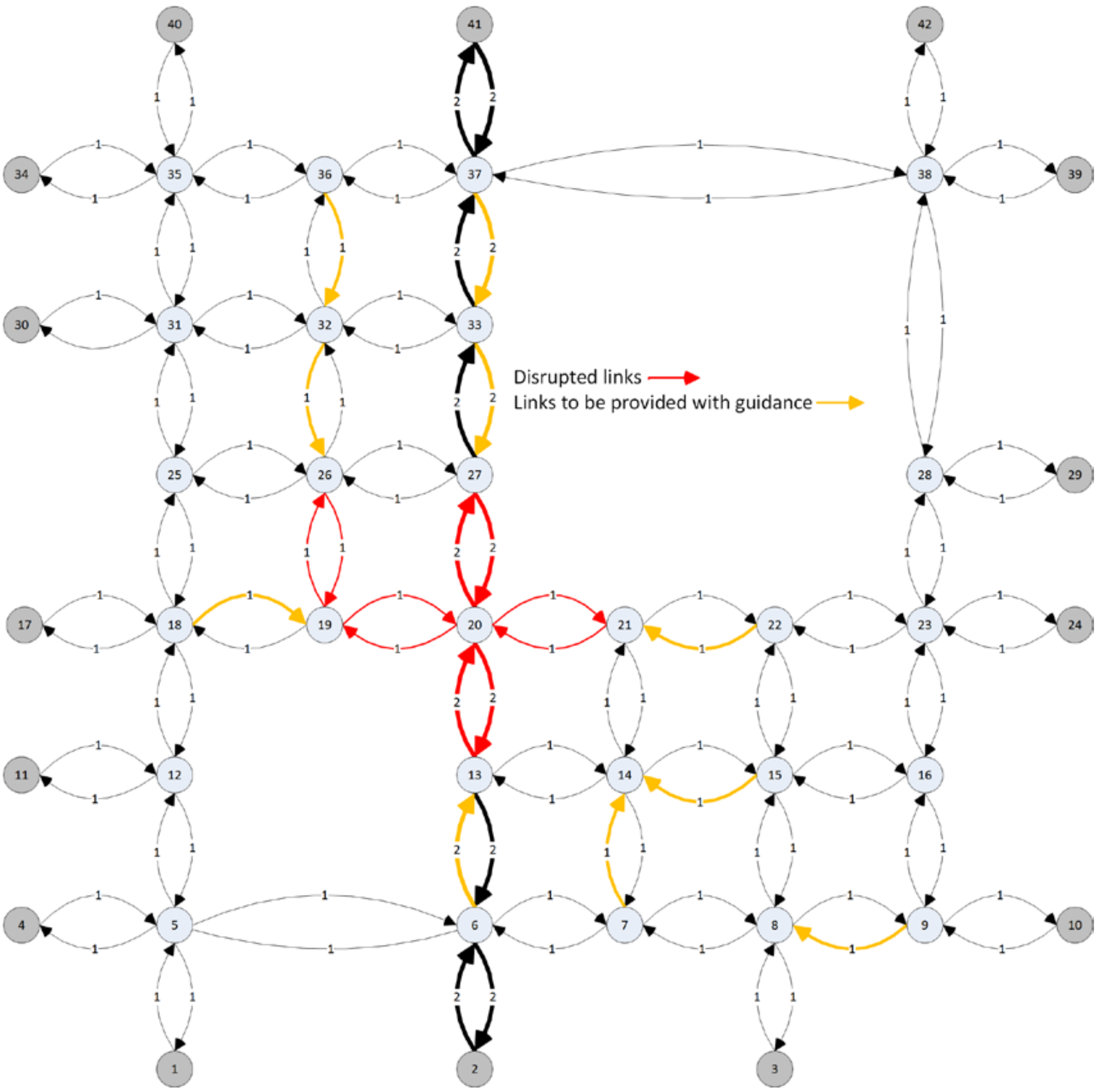
Links 20↔27, 20↔13, 19↔20, 20↔21 and 19↔26 are assumed to need long-term restoration due to damage from a disaster. As it can be seen in Figure 5, they are geographically concentrated in the centre of the network. Therefore, the total travel time of the network increases substantially to 47816 hours due to loss of operationally high capacity and topologically central links in the network. This figure will be much higher (75142) before the post-disaster equilibrium is reached (bottom of the road network performance curve in

Figure 2). The method proposed suggests providing road users with guidance information on links 33→27, 18→19, 22→21, 7→14, 6→13, 15→14, 37→33, 32→26, 36→32 and 9→8. It also suggests communicating the diversions shown in Table 5. As can be seen in Table 5, with this information, road users can divert earlier before they reach the disrupted links which would be beneficial not only to themselves but also for the whole performance of the network following the disaster.

Table 5: Guidance and diversions needed to be communicated on the node 6 (intersection) for scenario B

Source	Destination	Path
6	42	6 -> 7;7 -> 14;14 -> 21;21 -> 22;22 -> 23;23 -> 28;28 -> 38;38 -> 42;
	39	6 -> 7;7 -> 14;14 -> 21;21 -> 22;22 -> 23;23 -> 28;28 -> 38;38 -> 39;
	40	6 -> 5;5 -> 12;12 -> 18;18 -> 25;25 -> 26;26 -> 32;32 -> 31;31 -> 35;35 -> 40;
	34	6 -> 5;5 -> 12;12 -> 18;18 -> 25;25 -> 31;31 -> 35;35 -> 34;
	30	6 -> 5;5 -> 12;12 -> 18;18 -> 25;25 -> 31;31 -> 30;
	17	6 -> 5;5 -> 12;12 -> 18;18 -> 17;
	41	6 -> 5;5 -> 12;12 -> 18;18 -> 25;25 -> 26;26 -> 27;27 -> 33;33 -> 37;37 -> 41;

Figure 5: Critical links to be provided with guidance in scenario B



5. Concluding remarks

In this paper, we argue that the process of transition in link demand from a pre-disaster state to post-disaster state in which capacity of the road network is reduced can be achieved smoothly through guiding mechanisms such as Variable Message Signs (VMS). We propose a new method that identifies links where guidance could be provided to accelerate the transition from pre-disaster to a post-disaster equilibrium state.

These links should be considered as the most critical ones since they experience the most variation (reduction) in traffic volumes during pre-disaster equilibrium condition compared to a post-disaster equilibrium state. The proposed method for identifying both these critical links and optimal diversions plans to be communicated to road users were assessed on a test network using two different scenarios. In general, the proposed method could be exploited for traffic management as an essential recovery procedure during the aftermath of a disaster for improving road network resilience.

We also acknowledge that the proposed model in this paper only provides the preliminary steps for facilitating traffic flow during the recovery phase of disaster management while traffic is moving towards a new equilibrium. Hence, future work should take into account factors that we did not consider here. For instance, this model does not estimate and compare between how long transition from a pre-disaster to post-disaster equilibrium state takes with and without applying the guidance. Such estimation relies on two aspects of human behaviour: how much users follow the provided guidance and how their behaviour differs during the recovery period. The former requires stochastic modelling to cope with uncertainties, and the latter needs to consider multi-day effect. That is, both require empirical data for improving estimations. In addition, it is recommended to consider stochastic traffic assignment for the future work since it better captures the uncertainties in the human route choice behaviour.

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