Observing and reconstructing aggregated dynamic route choice patterns for large-scale networks

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ABSTRACT

This paper observes aggregated route choice patterns (i.e. vehicle distance traveled in different roadway classes, regional split ratios and average trip lengths) in a large scale mixed urban/freeway system through an extensive data set of 20,000 taxis in Shenzhen, China. It also reconstructs the aggregated patterns through shortest path algorithm that is based on various travel cost functions. We replace each observed trajectory with a shortest path that connects the same origin and destination points, and reproduce aggregated variables. We observe that link-level and regional interpretation of travel cost results in similar aggregated patterns. These results can enhance parsimonious network models and lead to better traffic predictions for large-scale congested networks.

INTRODUCTION

Large-scale traffic modeling and macro-scale management strategies remain a big challenge partly due to unpredictability of choices of travelers (e.g. route, departure time and mode choice). While there is strong understanding and vast literature of route choice modeling, there is no rigorous efforts, to the authors' knowledge, that investigates aggregated patterns (e.g. traffic load on freeway and urban subnetworks) that result from such modeling. Most of the analysis at the network level is based on simplistic models or simulations, (i) which are difficult to calibrate and (ii) require a large number of input variables/parameters, that might not be observable with the current available data. In addition, there is not enough explanation on if and how people adapt their choices with respect to dynamic traffic conditions in the network, and how all this changes network traffic properties.

Route choice models are essential to forecast travelers' behavior under hypothetical scenarios, and most importantly to predict traffic conditions in transportation networks. Although modeling route choice is quite challenging, given the complexity of human behavior and uncertainty about travelers' perceptions, they are essential part of dynamic traffic assignment (DTA) models that are expected to accurately predict traffic conditions in transportation networks. For instance, dynamic user equilibrium (DUE) assumes that travelers have the perfect knowledge of travel costs along the network, and choose the routes that minimize their travel costs. DUE state can be reached through the repeated implementation of shortest path algorithm over the iterations particularly in simulation-based DTA models (1). On the other hand, in dynamic stochastic user equilibrium (DSUE), travelers are assumed to have imperfect knowledge of travel costs, and choose the routes that minimize their perceived travel costs. While shortest path algorithm would be the straightforward way to establish DUE conditions, there is a vast literature of discrete choice models that could be exploited to reach DSUE conditions. The most appreciated models are Multinomial Logit, C-Logit (2), Path Size Logit (3), Link-Nested Logit (4), Multinomial Probit (5) and Error-Component model (6). Calibration of the above models is a challenging task because of computational complexity, lack of detailed data and occasionally overfitting of the parameters. Nevertheless, nowadays massive real-time data from multiple sources allow us to revisit and reformulate previous models to describe more accurately realistic congested traffic conditions.

The objective of this work is dual. On the one side, we utilize a unique dataset from a large number of probe vehicles that provide GPS location every few seconds in a megacity in China. We investigate if consistent empirical observations for dynamic route choice patterns can be made. Secondly we investigate how well the network level aggregated patterns can be estimated through shortest path or DUE assumptions. Obviously, not all travelers choose the shortest path to go from the origin to the destination point due to either lack of information or uncertainty of perception. However, the question is; what is the cost of using shortest path assumption when we aggregate the results from many origin-destination pairs? Transportation networks, by design, consist of urban motorways, expressways, large arterials, local streets, etc. Normally, long trips are expected to be bound to higher category roads, while short trips may use the local, finer-meshed network that can be continuously approximated. In this work, we exploit a very detailed GPS dataset of 20,000 taxis from Shenzhen, China, and we aim at exploring traffic load in different components of the transportation network. Aggregated route choice patterns could also be employed in order to improve predictive power of network traffic models.

The literature on parsimonious network traffic models is quite recent due to lack of detailed data (simulation efforts or empirical data at the static level have been assessed in the past). It was observed from empirical data in downtown Yokohama (7) that by spatially aggregating the highly scattered plots of flow vs. density from individual detectors, the scatter almost disappeared and a well-defined Macroscopic Fundamental Diagram (MFD) exists between space-mean flow and density. Real-time large-scale traffic management strategies, e.g. perimeter control (8, 9, 10, 11, 12, 13); gating (14, 15) that benefit from parsimonious models with aggregated network dynamics, provide promising results towards a new generation of smart hierarchical strategies. However, control strategies that require prediction of future traffic conditions face certain limitations regarding the route choice behavior in the multi-region urban network; travelers' reaction and adaptation to new management strategies is not considered in the control design. Yildirimoglu and Geroliminis (16) tackle this problem and establish equilibrium conditions in a multi-region urban network with MFD dynamics. They assume travelers have the knowledge of average traffic conditions in the subnetworks (e.g. city center, periphery roads, etc.), and choose the routes that minimize their approximate travel cost. In that respect, they establish DSUE conditions in the traffic network. Yildirimoglu et al. (17) extends this work to a route guidance strategy where travelers are forced to cooperate with each other in order to reach system optimum conditions. This paper aims for providing physical evidence from real data and exploiting aggregated route choice patterns in a large scale mixed urban/freeway network.

The remainder of the paper is organized as follows; in the next section, we introduce the GPS dataset from 20,000 taxis in Shenzhen. In the following section, the methodological framework for estimating aggregated dynamic route choice patterns is elaborated, and the results are presented. Finally, last section concludes the paper with future work directions.

DATA ANALYSIS

The data set consists of GPS tracks of around 20,000 taxis in a fast growing Chinese mega-city; Shenzhen. The rapid investment created one of the fastest-growing cities in the world with a population close to 11 million and, as expected, large congestion problems both in the urban and freeway system of the city. The network structure includes 28647 nodes and 35099 links, out of which 3354 are freeway links. The data set consists of trips (on the same day) from taxis equipped with a GPS sensor that stores its location every 10-40 seconds. For every GPS point, it is also known whether the taxi carries a passenger or not, which allows us to distinguish between trips with and without passengers. Assuming that taxi passengers follow routes similar to regular cars in the network, we only focus on taxi trips with passengers. Even if taxi drivers might seek non-standard paths, we expect that speed estimates based on taxis with passengers are a good representation of all vehicles and that aggregated patterns are not influenced much by local low-level route choices.

In order to identify traffic conditions in the network, we first map-match GPS observation with the closest link in the network, and estimate the link speeds. To make a similar analogy with (16), we also aggregate the GPS observations that are matched with urban links inside the 1x1 km regions and compute the average speed for the urban components inside them. Following equations present the estimated space-mean link and region speed, respectively.

$$\upsilon_l(t) = \sum_{j \in \mathcal{J}} s_j(l,t) / \sum_{j \in \mathcal{J}} h_j(l,t) \tag{1}$$

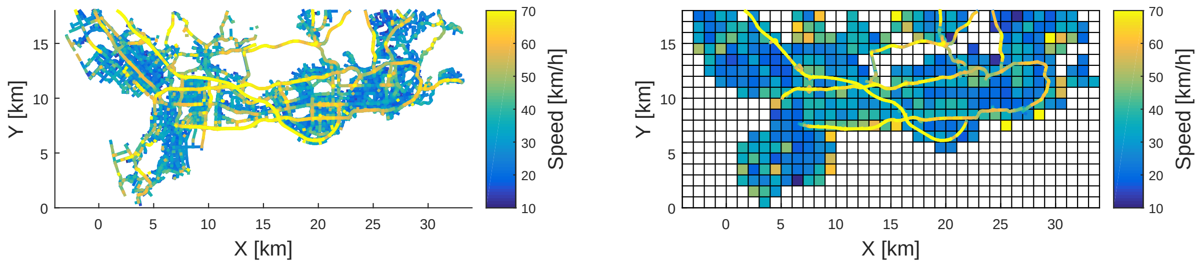
$$\upsilon_r(t) = \sum_{j \in \mathcal{J}} s_j(r,t) / \sum_{j \in \mathcal{J}} h_j(r,t) \tag{2}$$

where $\mathcal{J}$ is the set of all journeys, $s_j(l,t)/s_j(r,t)$ and $h_j(l,t)/h_j(r,t)$ are respectively the distance traveled and the time spent by journey j in link l / in urban region r at time period t. Note that in case there is no GPS observation on a link, the corresponding value is replaced with the average speed of neighboring links.

Figure 1(a) depicts the average speed in all the links throughout a 24-hr period (i.e. link-level matching, Eq. 1 applied for all links), while Figure 1(b) presents the average speed in the freeway links and 1x1 km cells that represent urban components of the network (i.e. regional matching, Eq. 1 and 2 applied for freeway links and urban cells, respectively). Note that, in the regional matching case, all urban links inside the same cell are assigned the same average speed. Clearly, the speed on freeway links is higher than the speed on surrounding urban links in both estimation methods, which points to the hierarchy of roads in mixed urban/freeway systems. Note that the average speed calculation could be done for smaller time periods, e.g. 1-hr. Figure 1(a) and 1(b) represents only the time-independent average traffic conditions in a day.

METHODOLOGICAL FRAMEWORK

In this section, we create aggregated route choice patterns through individual trip trajectories, and attempt to reconstruct them with shortest path results. In other words, we compare the patterns identified through the observed paths and the shortest paths that result from a few different travel cost considerations. The aggregated patterns we are interested in include average distance crossed in certain subnetworks, regional split ratios that define the proportion of accumulation aiming for a particular neighboring subnetwork and distance traveled in urban/freeway systems. The former two variables are strongly related with dynamic equations of MFD modeling, which will be further discussed in the next subsection. In addition, they are very important for developing mixed



(a) (b)

FIGURE 1: (a) Speed in all links, (b) speed in urban regions and freeway links.

urban/freeway control strategies, such as integrated corridor management (18) or mixed traffic signal/ramp metering schemes (9).

Travel cost functions that we consider in the shortest path calculation are;

- distance (regardless of hierarchy of roads)
- estimated travel time (a single constant speed for each speed limit zone, abbreviated as est. tt.)
- time-independent regional travel times (average values over 24-hr, abbreviated as reg. tt.)
- time-independent link travel times (average values over 24-hr, abbreviated as link tt.)
- time-dependent regional travel times (average values every 1-hr, abbreviated as dyn. reg. tt.)
- time-dependent link travel times (average values every 1-hr, abbreviated as dyn. link tt.)

The comparison of time-independent (static) and time-dependent (dynamic) cost scenarios may reveal the importance of dynamic traffic conditions in the network, and it may explain if and how people adapt to varying traffic conditions. In time-dependent cost scenarios, we employ the travel costs that correspond to departure time of trips in the shortest path calculation. On the other hand, the difference between regional and link-level matching may expose the significance of traveler perception regarding the travel costs along the network. Note that the number of observations per link especially in the urban system can be quite low, which may lead to under- or over-estimated link travel times. Regional matching puts all the measurements in the urban system together and assigns the smoothed average value to the links inside the same area.

MFD Modeling

Let us assume that the urban network is partitioned into several regions with well-defined MFDs. Let $Q_I(t)$ [veh/s] denotes the exogenous traffic flow demand generated in region I, $N_I(t)$ [veh] be the vehicle accumulation in region I. The traffic flow conservation equations are as follows:

$$\frac{\mathrm{d}N_I(t)}{\mathrm{d}t} = Q_I(t) - \sum_{H \in \mathcal{V}_I} M_I^H(t) + \sum_{H \in \mathcal{V}_I} M_H^I(t) \tag{3}$$

where $\mathcal{V}_I$ is the set of regions that are directly reachable from region I, i.e. adjacent regions to region I including region I itself. $M_I^I(t)$ [veh/s] would be the *internal trip completion rate* for accumulation in I with destination I (without going through another region), while the *transfer flow* for accumulation in I to neighboring region H is denoted by $M_I^H(t)$ [veh/s].

Internal trip completion rates ($I = H$) and transfer flows ($I \neq H$) are estimated corresponding to the ratio between accumulations as:

$$M_I^H(t) = \theta_{IH}(t) \cdot \frac{F_I\big(N_I(t), \sigma(N_I(t))\big)}{L_{IH}(t)} \tag{4}$$

where $F_I(\cdot)$ [veh.m/s] is the production MFD of region I (i.e. the total distance traveled per unit time in the region) that is a function of the region accumulation, $N_I(t)$, and the link density *heterogeneity* across all region I links, $\sigma(N_I(t))$. Moreover, $L_{IH}(t)$ [m] is the average trip length corresponding to transfer trips from region I to its neighbor region $H \in \mathcal{V}_I$, and $\theta_{IH}(t)$ is the percentage of accumulation in region I going through neighboring region H; hence $\sum_{H \in \mathcal{V}_I} \theta_{IH}(t) = 1$. Modeling the region link density heterogeneity, i.e. $\sigma(N_I(t))$, is investigated in (13). Previous work assumes $L_{IH}(t)$ and $\theta_{IH}(t)$ are constant permanently or over the prediction horizon, and identify the control actions by respecting this assumption. In this paper, we aim for providing the average trip length $L_{IH}(t)$ and split ratio $\theta_{IH}(t)$ using the available OD demand and shortest path assignment. This new piece of information could lead to a better design of traffic control/management strategies.

Aggregating route choice patterns

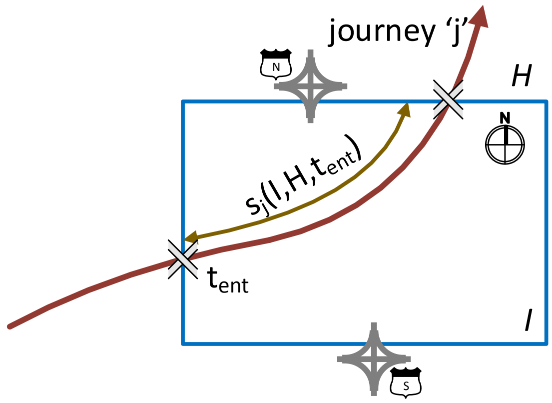
Let us assume that a rectangular urban region I (presented in Figure 2(a)) is connected to other homogeneous regions through its edges. In addition, there are freeway connections at the boundary of the region, which might be considered as a separate subnetwork. Therefore, $\mathcal{V}_I$ is a set of neighboring regions, freeway subnetwork and region I itself. The following formulas estimate the average trip length $L_{IH}(t)$ and split ratio $\theta_{IH}(t)$, respectively.

$$L_{IH}(t) = \sum_{j \in \mathcal{J}} s_j(I, H, t) \;/\; \sum_{j \in \mathcal{J}} \mathbf{1}_{\mathcal{R}^+}(s_j(I, H, t)) \quad H \in \mathcal{V}_I \tag{5}$$

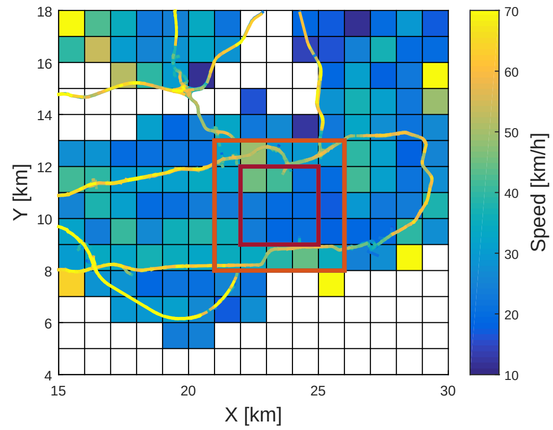
$$\theta_{IH}(t) = \sum_{j \in \mathcal{J}} \mathbf{1}_{\mathcal{R}^+}(s_j(I, H, t)) \;/\; \sum_{j \in \mathcal{J}} \mathbf{1}_{\mathcal{R}^+}(s_j(I, t)) \quad H \in \mathcal{V}_I \tag{6}$$

where $s_j(I, H, t)$ is the distance crossed by journey j in region I till the boundary of region H (see Figure 2(a)), and $\mathbf{1}_{\mathcal{R}^+}(.)$ is an indicator function with value 1 in case $(.) \in \mathcal{R}^+$, 0 otherwise. In other words, Eq. 5 represents the average distance for all vehicles that cross a non-negative distance in region I to go to neighboring region H, while Eq. 6 indicates the portion of vehicles going to region H among all vehicles that cross a non-negative distance in region I. Note that $s_j(.)$ can be coming from the observed GPS tracks or shortest path trajectories depending on the scenario analyzed.

In addition to route choice parameters needed in MFD modeling, one can estimate vehicle distance traveled (VDT) in urban and freeway systems. This information allows us to define the traffic load in different components of the mixed urban/freeway system. Let us denote $\mathcal{L}_F$ and $\mathcal{L}_U$ the set of freeway and urban links in the network, respectively. The following formula compute



(a)



(b)

FIGURE 2: (a) a trajectory through the area of interest, (b) Study area.

VDT in freeway and urban system.

$$dt_f(t) = \sum_{j \in \mathcal{J}} \sum_{l \in \mathcal{L}_f} s_j(l,t) \tag{7}$$

$$dt_u(t) = \sum_{j \in \mathcal{J}} \sum_{l \in \mathcal{L}_u} s_j(l,t) \tag{8}$$

RESULTS

In order to produce the route choice parameters required in MFD modeling, we choose an area I of 3x3 km (presented with red lines in Figure 2(b)). The area in the west of the square is the major city center with strong trip attractions and high level of congestion. Note that there are two freeway connections from the chosen urban area; one in the north, one in the south. For the sake of simplicity, we consider the vehicles that take the freeway via these interchanges as one single traveler group. This assumption can easily be relaxed later in order to calculate inflows to specific freeway junctions. Although a detailed partitioning algorithm is not yet conducted, we assume the chosen area is homogenous enough to assign MFD properties. We also assume there are 4 other homogeneous urban regions connected to the chosen area through its edges in 4 directions (i.e.north, south, west, east). Therefore, $\mathcal{V}_I$ is a set of six components; 4 neighboring regions, freeway subnetwork and region I itself. In other words, we define 6 traveler groups with distinct choices; heading for one of 4 neighboring regions, taking the freeway in the north or south, and finishing the trip inside the region. We apply Eq. 5-6 for all GPS trajectories and shortest paths that result from a set of travel cost definitions described in previous section. Since all OD pairs in the whole data set have been considered in the analysis, the number of trips that actually cross region I can be quite different for the observed trajectories and shortest path results. This might significantly affect the accuracy of estimated parameters.

Figure 3 introduces observed and estimated (or reconstructed) split ratios and average trip lengths. Note that solid curves in Figure 3 represent observed trajectories, while dashed curves are estimated with shortest path trajectories. Travel costs that have been tested here are time-dependent regional and link travel times. Not all but certain components of split ratios and average trip lengths reveal a strong dynamic behavior, see for example $\theta_{I,inside}$ (the fraction of trips that finish within the region) and $L_{I,west}$ (the average trip length for trips exiting through the west boundary), and these patterns have been roughly followed by estimated parameters. Note that, between 11h and 13h, in dynamic link travel time case, $L_{I,north}$ estimations are not very consistent with neighboring time periods (see Figure 3(d)). This is again probably due to low sampling and inaccurate travel time estimation for certain links. The network is much less dense in the northern part of the square and few destinations hit towards this direction (less than 3% as shown in Figure 3(a) and 3(c)).

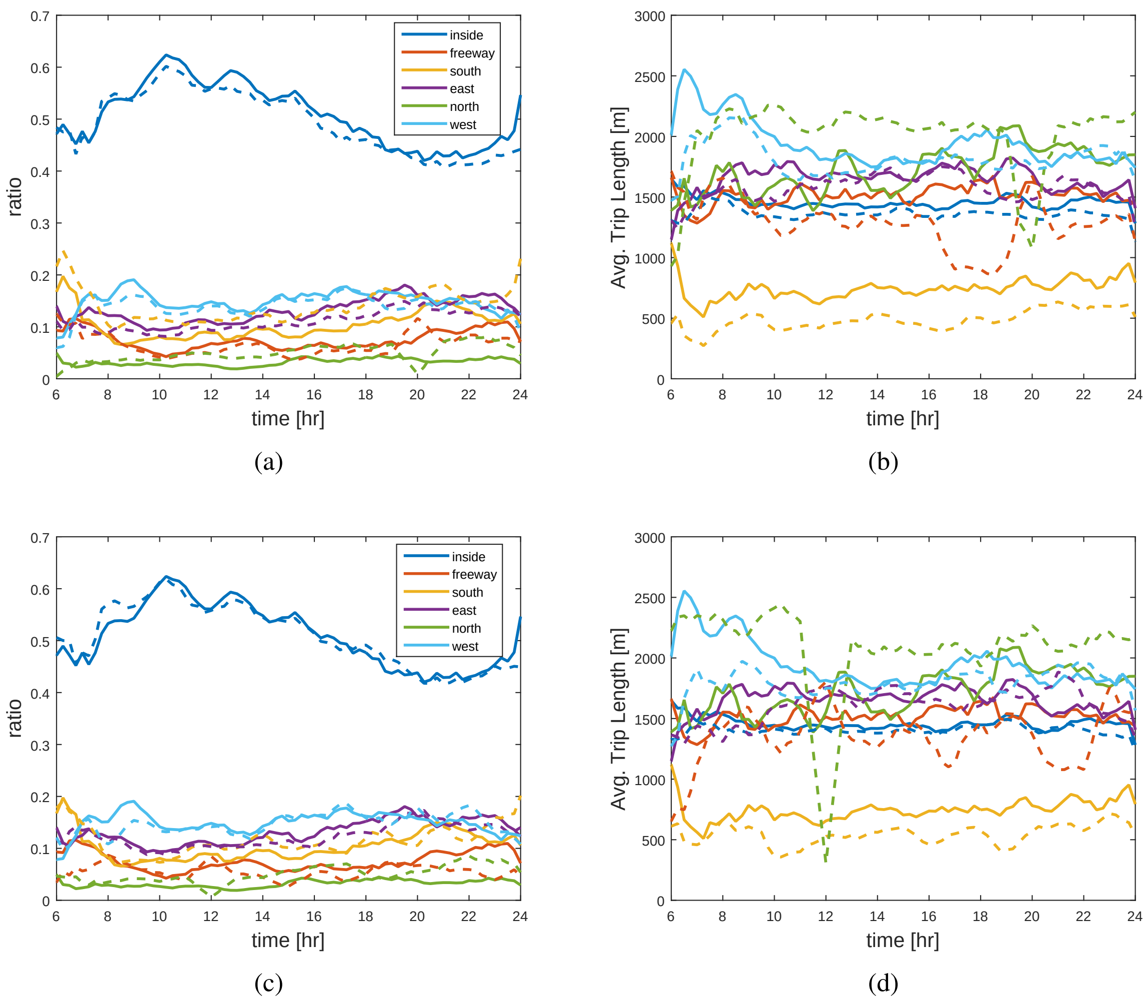
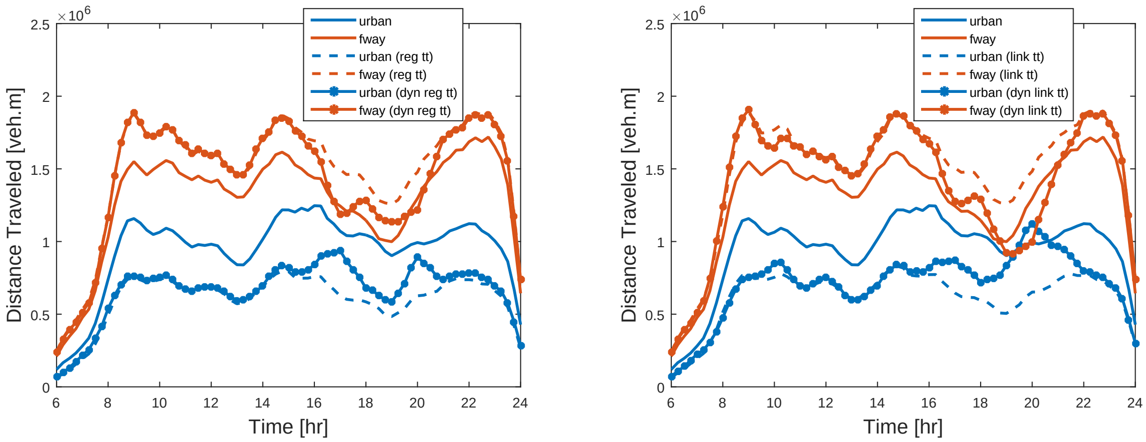


FIGURE 3: (a) Split ratios with dynamic regional travel time, (b) average trip lengths with dynamic regional travel time, (c) split ratios with dynamic link travel time, (d) average trip lengths with dynamic link travel time.

In order to calculate dt_f and dt_u values, we choose an area of 5x5 km with both freeway and urban links (see the orange lines in Figure 2(b)). Note that previously chosen area of 3x3 km does not include any freeway links, and therefore it is not suitable for the traffic load analysis. We apply Eq. 7 and 8, and calculate the observed and estimated VDT values using actual and shortest path trajectories, respectively. Figure 4 depicts the resulting curves for static/dynamic regional/link travel times. Note that, in all scenarios, freeway load has been overestimated (for about 10%), and consequently urban system use has been underestimated. In fact, one of the major freeways present in this analysis is a toll freeway. Therefore, travel cost on this specific freeway section should be a certain combination of toll fee and travel time. However, this analysis will be reported in an extended version of this article; this issue requires further investigation. Another interesting observation is the similarity between time-dependent and -independent scenarios. Except the afternoon peak period, both scenarios produce quasi-identical results. This implies that relative speeds in urban and freeway systems remain approximately constant throughout the day except the afternoon peak.



(a) (b)

FIGURE 4: (a) VDT with regional travel times, (b) VDT with link travel times

Table 1 presents the mean absolute errors (MAE) for the estimation of variables presented in Figure 3-4. For each subcomponent (e.g. traveler groups for θ_{IH} and L_{IH}, urban and freeway subsystems for VDT) and each observation (e.g. time periods for θ_{IH}, L_{IH} and VDT), we simply calculate the absolute difference between the observed and estimated (or reconstructed) variables, and compute the average of all to report MAE. In overall, estimations that are based on distance or estimated travel time produce quite inaccurate results compared with other travel cost definitions. Although time-dependent regional travel time produces slightly better results than its time-independent version in case of θ_{IH} and L_{IH} estimation, dynamic link travel time fails to improve the results of its static counterpart. The probable reason behind it might be the lack of sufficient number of observations for certain links. Note that the best estimation has been achieved with time-independent link travel time computation for both θ_{IH} and L_{IH}. In case of dt_f, dt_u estimation, dynamic cost definitions produce better results than their static counterparts, and the best results have been obtained with time-dependent link-level travel times.

DISCUSSION

This paper is a first attempt, to the authors' knowledge, to identify and to reproduce aggregated route choice patterns in a mixed urban/freeway environment. An extensive GPS data set of 20,000 taxis from Shenzhen, China make it possible to monitor, understand and analyze aggregated route choice patterns. The variables that we focus on in this paper include regional split ratios, average trip lengths, VDT in different systems and urban/freeway shares. Note that the former two variables

TABLE 1: Mean Absolute Error (MAE) of estimated parameters

	Travel Cost					
	distance	est. tt	reg. tt	link tt	dyn. reg. tt	dyn. link tt
θ_{IH}	0.038	0.042	0.023	0.015	0.019	0.015
L_{IH} [m]	395	372	199	191	195	201
VDT [veh.m.1e5]	4.35	2.96	2.30	2.20	1.86	1.77

can be embedded in MFD dynamics, and may lead to a better traffic prediction model. However, testing of such hypothesis requires data from both loop detectors and probe vehicles, which is not available in this case. The main objective of this work is to reproduce the aggregated patterns through shortest path assumption. We replace each observed trajectory with a shortest path that connects the same origin and destination points, and reproduce aggregated variables that result from different travel cost considerations. Although there are certain problems (e.g. overestimation of freeway use) regarding the shortest path assumption, the results are quite promising.

Future work should focus on discrete choice models instead of shortest path algorithms. A better route choice estimation at individual trip level will definitely lead to better estimation of aggregated route choice patterns. Discrete choice models may also allow us to distinguish between toll freeways and others in an explicit way. This might overcome the issue of overestimation on freeway load. Another future research direction is to test the accuracy of aggregated route choice variables regarding MFD dynamics. Previous work in literature estimates average trip lengths through loop detector data. In other words, outflow value identified through loop detectors has been employed to estimate average trip lengths (see Eq. 4). However, in this paper, we propose trajectory-based calculations. The integration between these two should be analyzed for a real network where both probe vehicle and loop detector data are available (this is not the case in Shenzhen).

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