Data envelopment analysis (DEA) based transit route temporal performance assessment: a pilot study

Khac-Duong Tran¹, Ashish Bhakar², Jonathan Bunker³, Boon Lee⁴

¹,²,³ School of Civil Engineering and Built environment, Science and Engineering Faculty, ⁴QUT Business School, School of Economics and Finance, Queensland University of Technology (QUT), 2 George Street, Brisbane, Australia

Email for correspondence: khacduong.tran@hdr.qut.edu.au

Abstract

The transit agencies aim to allocate limited resources properly and maximise ridership. Measuring the performance of individual transit routes within a transit system plays a critical role in finding operational problems and increasing transit ridership.

The Data Envelopment Analysis (DEA) method has been employed to compare the performance of different units (e.g., transit operators, transit lines/routes within a transit system). This paper first reviews the application of DEA for transit performance evaluation. Thereafter, with a case study on a bus route in Brisbane, a pilot study is conducted to discuss the application of DEA for temporal performance evaluation (service effectiveness) of a transit route considering number of services and travel time as inputs and transit work and on-time performance as outputs.

Keywords: Data envelopment analysis, smart card, Automatic fare collection, transit performance evaluation.

1. Introduction

The transit agencies aim to increase ridership, for instance Queensland government, Australia has an official targets to increase the transit ridership in South East Queensland from 7% in 2006 to 14% in 2031 (connecting-seq-2031-finalised). Agencies have limited resources and increasing financial burden. Therefore, smart utilisation of the limited resources is the need of the hour.

Measuring the performance of individual transit routes within a transit system plays a critical role in finding problems in the transit system design, operation and control, and increasing transit ridership. However, evaluating the performance of individual transit routes is a complex procedure because multiple objectives and multiple input and output variables relate to this procedure (details in section 2). The transit agencies thus need to develop and exploit tools that can support them to evaluate the efficiency of the transit routes performance, identify the key factors leading to the inefficiency, and make rational decisions for planning, operations and management of their network. Data Envelopment Analysis (DEA) method has been utilized widely for comparing the performance of different transit systems or different transit routes as production units (Chu, Fielding, & Lamar, 1992; Georgiadis, Politis, & Papaioannou, 2014; Sheth, Triantis, & Teodorović, 2007; Viton, 1997, 1998; M.-M. Yu & Fan, 2009). However, due to the simple transit data collected through manual survey, the application of DEA models to measuring the performance of individual transit routes is fairly limited.

The objective of the paper is twofold. It first, critically reviews the applications of DEA model to evaluating the performance of transit systems. Thereafter, with a case study on Brisbane, a pilot study is conducted to evaluate a DEA based temporal performance of a bus route (Route 111). Recently, the availability of smartcard based automated fare collection systems
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(AFC) in the transit sector has provided a valuable opportunity for estimating transit performance indicators in greater detail (Trépanier, Morency, & Agard, 2009). In this paper we exploit AFC data to generate the indicators needed for the aforementioned case study.

The remaining of the paper is organised as follows: section 2 introduces the concepts of transit performance; section 3 provides information about the applications of DEA model to measuring the performance of transit systems, and individual transit routes within a transit system. The DEA approach is presented in section 4. The case study on a Brisbane bus route is presented in section 5 and finally the paper is concluded in section 6.

2. Transit performance concepts

The performance of a given transit system as well as transit route can be distinguished into the three dimensions: **technical efficiency** (also termed as **cost efficiency**), **operational effectiveness** (also termed as **cost effectiveness**), and **service effectiveness** (see Figure 1).

- **Technical/cost efficiency** represents the process through which service inputs are transformed into outputs. This means that a transit agency will invest capital in the transit vehicles, fuel, the information systems, employees, maintenance, and other costs (inputs). This investment will produce a certain service for a community such as vehicle-kilometres, seat-kilometres, and seat-hours which forms the outputs. An agency is considered efficient if it can reduce the inputs to produce a fixed amount of outputs, or it can increase the outputs while using the similar or less inputs.

- **Operational/cost effectiveness** indicates the relationship between service inputs and consumed services. A transit agency spends money to offer its service, and a number of passengers (per day or week) consume its service. Transit agency will achieve higher cost effectiveness, if it increases the ridership without increasing total cost of producing the service.

- **Service effectiveness** examines the relationship between produced outputs and consumed service or how well a service offered by operators can be consumed by a community (Georgiadis et al., 2014), which means that not all of the service offered by a transit agency (measured by vehicle-kilometres, seat-kilometres, and/or seat-hours) will be used by a community. If it attracts more passengers without increasing the service, or reduces the service but still serves the similar number of passengers, it will be more effective. Note: The case study (section 5) applied in this paper evaluates service effectiveness.

![Figure 1. Framework for a transit performance concept model (Fielding et al. (1985))](image-url)
Figure 1 illustrates that there are a number of uncontrollable variables (population density, accessibility, parking space availability, car ownership) influencing the actual service consumption of a community with regard to the effectiveness perspective. Also, concerning the efficiency component, external factors (traffic conditions, location of transit stops) significantly affect the produced service.

3. The application of DEA approach for transit performance evaluation

Measuring the performance of urban transit systems with regard to the efficiency and the effectiveness is a major challenge to transit agencies, as multiple factors simultaneously influence the operation of any public transport system. Fielding et al. (1985) used the cluster analysis to construct 12 peer groups of fixed-route urban transit. They then analysed the variance and discriminant among the peer groups in terms of operating characteristics to build up a decision tree typology that is an intellectual device for clarifying the performance similarities as well as differences among transit agencies. This approach provided the basic for developing the Irvine Performance Evaluation methodology (IPEM), which subsequently was used by some researchers to study the performance of transit agencies like Perry et al. (1986), Yu (1988) and Fielding et al. (1988). However, the IPEM statistics is a cumbersome method for evaluating transit performance. It does not provide a single overall measure of transit performance (Chu et al., 1992).

To overcome this drawback, Chu et al. (1992) applied the DEA model to measure the efficiency and effectiveness of public transit agencies in the United States (US). Based on the results obtained, the authors reinforced the notion of Hatry (1980) that in measuring the performance of transit agencies, efficiency should be evaluated separately from effectiveness. Thereafter, many researchers have used DEA models for transit performance analysis (Barnum, Tandon, & McNeil, 2008; Georgiadis et al., 2014; Karlaftis, 2004; Lao & Liu, 2009; Roháčová, 2015; Sheth et al., 2007; Tsamboulas, 2006; Viton, 1997, 1998).

Table 1 provides an overview of the application of DEA models to measuring the transit performance at both system and route levels. Here, the review is separated into two groups: the formers focuses on the performance of transit systems and the later focuses on the performance of individual transit routes/lines. The columns represent the DEA models used, number of Decision Making Units (DMUs), inputs and outputs selected for DEA models, time frame of data, and finally the findings.

As summarized in Table 1 most of the research focuses on evaluating the performance of different transit systems on yearly data (first group). Recently, few researchers have focused on evaluating the performance of individual transit routes within a system (Triantis, 2004). Comparing the performance of different transit systems plays a key role in determining the average operational efficiency of a transit system and problems related to the operation of the whole system, but cannot explore the problems related to the internal activities of each transit route. On the other hand, the performance evaluation of individual transit routes within a transit system substantially provides the transit agency the opportunity to understand its internal activities (Barnum et al., 2008; Benn, 1995), and then investigate the source of inefficiency. Possible actions then can be taken by transit agencies to optimize the operational efficiency of inefficient transit routes, and thus leads to performance improvement for the whole transit system. Evaluating the performance of individual transit routes therefore is of importance for optimizing the operation of transit route and system.

Most of the research has focused on technical efficiency and cost effectiveness. Researchers have evaluated the relationship between the technical efficiency and cost effectiveness and literature had contrary findings. Chu et al. (1992) suppose that these two dimensions of transit performance should be evaluated separately, while Karlaftis (2004)
claims that efficiency and effectiveness seem to be positively related. The correlation between technical efficiency and cost effectiveness should be further studies.

Limited research (Barnum et al., 2008; Lao & Liu, 2009) is on service effectiveness. This is because of the complexity in modelling the service effectiveness which is often based on the uncontrolled factors (such as living standards of the residents, quality of service with respect to passenger perception, parking space and private vehicle ownership). Moreover, the availability of the integrated data needed to the modelling is also hard to obtain.

The DEA model only provides a mean of estimation of DMUs’ technical efficiency (TE). To evaluate the factors (environmental variables such as socio-economic variables) affecting the efficiency level, a two stage process is adopted (Nolan (1996), Georgiadis et al. (2014)). Here, at the first stage the DEA model is applied to estimate the TE and thereafter, at the second stage a truncated regression model is applied to analyse the sensitivity of the TE values obtained in the first stage to those factors. However, the limitation of these studies is that they lack information on some potential uncontrollable variables such as structure of population, private vehicle ownership, and average income of residents. The environmental factors influencing the transit performance thus were not studied sufficiently.

The performance evaluation of individual transit routes within a transit system has drawn the attention of a few researchers (Barnum et al., 2008; Georgiadis et al., 2014; Lao & Liu, 2009; Roháčová, 2015; Sheth et al., 2007). However, due to the simple transit data collected through manual survey, the temporal and spatial performance of transit routes in those studies was not analysed sufficiently. For instance, the travel time is estimated through the operating speed which depends on the distribution of transit route in urban or suburban area. Most researchers use “passenger-km” as the output for evaluating the service effectiveness of transit route, while the corresponding input is “seat-km” representing vehicle passenger carrying capacity. “Passenger-km” was defined as the total number of passenger transmission of a route multiply by the total number of kilometres travelled by all the vehicles operating on the corresponding route during a weekday. “Passenger-km” thus does not reflect the service consumption accurately because it considers the total number of kilometres travelled by all the vehicles instead of the average route length travelled by passengers.

Regarding the above relationship between the vehicle passenger carrying capacity and the service consumption, Vuchic (2007) defined “transportation work” (\(w\)) as the number of transported objectives (\(u\)) multiplied by the distance (\(s\)) over which they are carried: \(w = u \cdot s\)

Based on the work of Vuchic, Bunker (2013) introduced “transit work” and “transit service work efficiency” of an individual transit service \(h\) along its route \(L\) with \(n\) segments constituting route \(L\). “Transit work” was the sum of the transit work performed along all consecutive segments along the transit route.

Transit work performed by service \(h\) along its route \(L\), given by (p-km):

\[
W_{h,L} = \sum_{i=1}^{n} P_{OB,h,i}s_i
\]

Where: \(s_i\) = length of segment \(i\)

\[
P_{OB,h,i} = \text{Passengers on board for service } h \text{ along segment } i
\]

\(n = \text{Number of consecutive segments constituting line } L \text{ traversed by service } h\)

It is clear that “transit work” comparing to “passenger-km” reflects the service consumption more accurately because “transit work” takes the actual route length traversed by passengers into account and reflects the actual vehicle’s loading level along the transit route.
Table 1: An overview of the application of DEA models in measuring the transit performance

<table>
<thead>
<tr>
<th>References</th>
<th>DEA model</th>
<th>DMUs</th>
<th>Inputs</th>
<th>Outputs</th>
<th>Time frame considered</th>
<th>The findings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obeng (1994)</td>
<td>DEA model</td>
<td>73 bus agencies in USA</td>
<td>Labour; Fuel; Fleet size</td>
<td>Vehicle- Miles</td>
<td>Annual data</td>
<td>Subsidies improve technical efficiency (TE) in approximately 75% of the transit systems studied</td>
</tr>
<tr>
<td>Nolan (1996)</td>
<td>DEA model (BCC-DEA) and Tobit model</td>
<td>25 mid-sized bus agencies in USA</td>
<td>Vehicle operated; Fuel; Labour.</td>
<td>Vehicle- Miles</td>
<td>Annual data</td>
<td>Average fleet age is significantly and negatively correlated with the TE measure. Operating subsidies can create significant and negative impacts on TE.</td>
</tr>
<tr>
<td>Kerstens (1996)</td>
<td>DEA model and Free Disposal Hull (FDH) DEA model</td>
<td>114 French urban transit companies</td>
<td>Vehicles; Employees; Fuel. Explanatory variables: Owner; Group; Linelength; Stoplength; Popdens; Vehage; Ctype; Cterm; Ssub; Tax.</td>
<td>Vehicle-Km; Seat-Km</td>
<td>Annual data</td>
<td>It confirms the important role of the alternatives among deterministic nonparametric approach for TE assessment, and the relevance of ownership and the harmful impact of subsidies.</td>
</tr>
<tr>
<td>Viton (1997)</td>
<td>Russel DEA model, with VRS + Weak Disposal</td>
<td>217 multi-mode motor-bus transit systems in USA</td>
<td>Average speed; Average Fleet age; Number of directional miles; The fleet sizes; Fuel; Labour hours for transportation, maintenance, admin, capital; Tires and material cost; Service cost; Utilities cost; Insurance cost.</td>
<td>Vehicle-miles; Passenger-trips.</td>
<td>Annual data</td>
<td>Public and private systems do not have an observed systematic efficiency difference. Around 80% of the sample is technically efficient. The extent of inefficiency in the industry is slight.</td>
</tr>
<tr>
<td>Viton (1998)</td>
<td>The Russell and Malmquist DEA models</td>
<td>183 US bus systems in 1988, and 169 systems in 1992</td>
<td>Average speed; Average fleet age; Number of directional miles; The fleet sizes; Fuel; Labour hours for transportation, maintenance, admin, capital; Tires and material cost; Service cost; Utilities cost; Insurance cost</td>
<td>Vehicle-miles; Passenger-trips; Vehicle-hours.</td>
<td>Annual data</td>
<td>Bus transit efficiency has improved slightly over the period. The proportion of technically efficient systems rose from 74% in 1988 to 82% in 1992. In most inefficiency category, there were proportionately...</td>
</tr>
<tr>
<td>Study</td>
<td>Model Assessed by</td>
<td>Number of Agencies/Systems in USA</td>
<td>Data Source</td>
<td>Cost Components</td>
<td>Revenue Vehicle Hours</td>
<td>Annual Data</td>
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<tr>
<td>Chu et al. (1992)</td>
<td>DEA model</td>
<td>86 bus agencies</td>
<td>Vehicle operating cost; Maintenance cost; General cost; Other expenses; Revenue vehicle hours; Population density; % of household with car; Subsidy passenger</td>
<td>Revenue vehicle hours; Unlinked passenger-trips</td>
<td>Annual data</td>
<td>Fewer systems in 1992 than in 1988.</td>
</tr>
<tr>
<td>Boilé (2001)</td>
<td>DEA model</td>
<td>23 bus agencies</td>
<td>The operating costs; Vehicle revenue hours</td>
<td>Vehicle revenue hours; Unlinked passenger-trips</td>
<td>Annual data</td>
<td>Systems that operate locally inefficiently may improve their service by using operation strategies. System that exhibit scale inefficiencies may be improved upon by identifying and dealing with external factors.</td>
</tr>
<tr>
<td>Karlaftis (2004)</td>
<td>DEA model and the Return to scale analysis.</td>
<td>256 transit systems</td>
<td>Total vehicles; Fuel; Total employees</td>
<td>Total annual vehicle-miles; Total annual ridership</td>
<td>Annual data (1990-1994)</td>
<td>Efficiency and effectiveness are positively related. Optimal scale of operation varies significantly and depends on the output specification selected and the performance dimension.</td>
</tr>
<tr>
<td>Tsambo-ulas (2006)</td>
<td>DEA model and Tobit regression model</td>
<td>15 European transit systems</td>
<td>Total vehicles; Total employees; Transit system characteristics: Population; Area.</td>
<td>Vehicle-Km; Passengers</td>
<td>Annual data (1990-2000)</td>
<td>Private systems are more efficient, while public systems are more effective. The transit systems appear to have experienced a certain growth during the examined time period.</td>
</tr>
<tr>
<td>Ayadi (2013)</td>
<td>DEA model and an econometric regression model.</td>
<td>12 urban transit systems in Tunisia</td>
<td>Total number of bus park; Number of staff; Annual amount of fuel consumed</td>
<td>Travelled Km</td>
<td>Annual data (2000-2010)</td>
<td>The annual technical efficiency (input orientation) is 92.44%. The average technical efficiency (output orientation) is 90.13%</td>
</tr>
<tr>
<td>Authors</td>
<td>Methodology</td>
<td>Routes/Geography</td>
<td>Performance Measures</td>
<td>Annual Data</td>
<td>Notes</td>
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<tr>
<td>Lao et al. (2009)</td>
<td>DEA model and geographic information system (GIS)</td>
<td>24 fixed bus routes in Monterey County, California, USA.</td>
<td>Operation time; Round trip distance; Number of bus stops; Commuters who use buses; Population 65 and older; Persons with disabilities</td>
<td>Total number of passenger. Total number of passenger.</td>
<td>For TE: 6 bus lines are technically efficient, 6 bus lines are fairly efficient (scores ≥ 0.6), and 12 bus lines are inefficient. For spatial effectiveness: 11 of them are technically efficient (scores ≥ 0.8) and 13 bus lines are inefficient.</td>
<td></td>
</tr>
<tr>
<td>Barnum et al. (2008)</td>
<td>DEA model</td>
<td>46 bus routes of a US transit agency</td>
<td>Seat kilometre (SK); Seat hours (SH); Population density; Population.</td>
<td>Ridership; Span of service; Average frequency; Maximum frequency; On-time performance.</td>
<td>The average weekday trips Comparing the performance of multiple bus routes of one transit agency. 20 bus routes became more efficient, 12 did not change, and 14 became less efficient.</td>
<td></td>
</tr>
<tr>
<td>Sheth et al. (2007)</td>
<td>Network DEA model</td>
<td>60 bus routes in Virginia, USA.</td>
<td>The provider node: Headway; Service duration; Costs; Number of intersections; Priority lanes. The societal variable: Number of accidents; Emissions; Noise pollution; Resources degraded. The environmental variables: Accessibility; Parking space availability; Population density; Connectivity; Comfort standards factor.</td>
<td>The provider node and inputs for the passenger node: Vehicle-mile; Schedule reliability; Average travel time. The passenger node: Passenger-mile.</td>
<td>The average weekday trips Capture the relationship among the supplier, the customer of the transportation service as well as the external and environmental variables related to the urban transit performance.</td>
<td></td>
</tr>
<tr>
<td>Georgiaidis et al. (2014)</td>
<td>DEA model and Bootstrap-ping techniques</td>
<td>60 bus routes in Greece.</td>
<td>Model 1: Length; Span of service; Vehicles. Model 2: Length; Span of service; Vehicles. Model 3: Revenue vehicle-km; Vehicles.</td>
<td>Revenue seat-km; Passenger</td>
<td>Annual data (2009-2011) There is not clear relationship between efficiency and operational effectiveness. Evaluating the transit route performance is more reliable when correcting for bias.</td>
<td></td>
</tr>
</tbody>
</table>
4. Data envelopment analysis (DEA)

Data envelopment analysis (DEA), as developed by Charnes, Cooper, and Rhodes (CCR) in 1978 and later modified by Banker, Charnes and Cooper (BCC) in 1984, builds on the frontier efficiency concept first elucidated in Farrell (1957). DEA is a non-parametric and empirical modelling based on linear programming and optimization. It is widely used to measure the relative efficiencies of production units (termed as Decision making units, DMUs) with multi-inputs and multi-outputs. Literature is abundant with its application in banking (Depren & Depren, 2016; Mohamed Shahwan & Mohammed Hassan, 2013), hospitals (Jat et al., 2013; Torabipour, Najarzadeh, Arab, Farzianpour, & Ghasemzadeh, 2014), schools (Agasisti, 2013; Rosenmayer, 2014), electricity (Andrade, Alves, Silva, & de Mello, 2014; Azadeh, Motevali Haghighi, Zarrin, & Khaefi, 2015), and transportation (Fancello, Uccheddu, & Fadda, 2014; Georgiadis et al., 2014; Lao & Liu, 2009; Zhao, Triantis, Murray-Tuite, & Edara, 2011).

The modelling process of DEA includes: a) identification of the production frontier (or isoquant) of a set of comparable DMUs. Within a set of comparable DMUs, those exhibiting the best use of inputs to produce outputs will be identified, and would form an efficient frontier; b) measures the level of efficiency of each DMU by comparing its production function with the production frontier (Cook & Seiford, 2009). The production function (technology) is described by the production possibility set $T$ of feasible output vectors $y$ producible from input vectors $x$:

$$ T = \{(x, y): y \text{ is feasibly produced from } x\} \tag{2} $$

The CCR model measures efficiency of a DMU relative to a reference technology exhibiting constant returns to scale (CRS) whereas the BCC model exhibits variable (increasing, constant, or decreasing) returns to scale (VRS) at different points on the production frontier (see Figure 2). These two basic DEA models play a crucial role in providing practitioners a non-parametric approach to evaluate the efficiency of DMUs with multi-inputs and multi-outputs.

In transit, due to capacity constrains (bus station capacity) the output (on time performance, transit work) might not have a constant increase by increasing the inputs (the size of the bus, service frequency etc.). Therefore the return to scale might not be constant. However for the current problem application (case study, section 5) we can consider CRS under the assumption that the system is operating below capacity. For broader application, we need to consider VRS so as to reflect the capacity constrain. The comparison of the results from CRS is beyond the scope of this paper. As this study utilises CRS, next section provides the details of the CRS model. Interested readers can refer to Coelli et al. (1998) for detailed understanding of CRS and VRS models.

Figure 2. Production frontier of CCR (CRS) and BCC (VRS) models
Data envelopment analysis (DEA) Based Transit Route Temporal Performance Assessment

**CCR model**

Suppose that each DMU \( j = 1 \ldots n \) uses \( m \) inputs \( x_{ij} (i=1 \ldots m) \) to generate \( s \) outputs \( y_{rj} (r=1 \ldots s) \), and the \( v_i, u_r \) are the variable weights of inputs and outputs respectively.

This method uses the known inputs and outputs of all DMUs in the given set of data to determine the efficiency of one member DMU \( j = 1 \ldots n \), which is assigned as DMU \( 0 \). The efficiency of DMU \( 0 \) is obtained by solving the following fractional programming problem \( n \) times, each DMU once.

\[
\max h_0 = \frac{\sum_{r=1}^{s} u_r y_{r0}}{\sum_{i=1}^{m} v_i x_{i0}} \tag{3}
\]

Subject to:

\[
\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1; \quad j = 1, \ldots, n
\]

\[
u_r, v_i \geq \epsilon > 0; \quad r = 1, \ldots, s; \quad i = 1, \ldots, m.
\]

Where \( \epsilon \) is a "non-Archimedean infinitesimal", which is smaller than any positive real number. This means that all variables are constrained to positive values.

The objective is to obtain the input and output weights \( v_i, u_r \) as variables that maximize the ratio of the DMU \( 0 \), the DMU being evaluated. The value of \( h_0 \) obtained from this formulation represents the efficiency score of the DMU \( 0 \). The constraints mean that \( h_0 \), the optimal value of \( h_0 \), should not exceed 1 for every DMU. In case \( h_0 = 1 \), this DMU places on the efficiency frontier (Tone, Cooper, & Seiford, 1999).

To solve this problem, the authors apply the theory of Charnes and Cooper (1962) to converted this fractional programming problem to the linear programming (LP) model with the changes of variables \( t (\sum_{i=1}^{m} v_i x_{i0}) = 1; \mu_r = tu_r \) and \( \vartheta_i = tv_i \). The above problem is replaced by the following equivalent:

\[
\max h_0 = \sum_{r=1}^{s} \mu_r y_{r0} \tag{4}
\]

Subject to:

\[
\sum_{i=1}^{m} \vartheta_i x_{i0} = 1
\]

\[
\sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} \vartheta_i x_{ij} \leq 0 \quad j = 1, \ldots, n
\]

\[
\mu_r, \vartheta_i \geq \epsilon > 0; \quad r = 1, \ldots, s; \quad i = 1, \ldots, m.
\]

The dual problem reproduced here for input-oriented model is as follows:

\[
\min \theta - \epsilon (\sum_{r=1}^{s} s_r^+ + \sum_{i=1}^{m} s_i^-) \tag{5}
\]

Subject to:

\[
\sum_{i=1}^{m} \lambda_i x_{ij} + s_i^- = \theta x_{i0} \quad i = 1, \ldots, m
\]

\[
\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{r0} \quad r = 1, \ldots, s
\]

\[
\lambda_j, s_r^+, s_i^- \geq 0, \quad \text{all } r, i, j; \quad \theta \text{ free}
\]

Where: \((s_r^+, s_i^-)\) are the output and input slack variables

In case of output-oriented model, the dual problem can be expressed as follows:

\[
\max \varphi - \epsilon (\sum_{r=1}^{s} s_r^+ + \sum_{i=1}^{m} s_i^-) \tag{6}
\]
Subject to: 
\[
\sum_{j=1}^{n} \lambda_j x_{ij} + s^-_i = x_{i0} \quad i = 1, \ldots, m
\]
\[
\sum_{j=1}^{n} \lambda_j y_{rj} - s^+_r = \varphi y_{r0} \quad r = 1, \ldots, s; \lambda_j, s^+_i, s^-_i \geq 0, \text{ all } r, i, j; \quad \varphi \text{ free}
\]

5. DEA based bus route temporal performance: a case study on route 111, Brisbane

With a case study on a single bus route 111 on Brisbane, Australia the paper explores the application of DEA for evaluating the temporal performance of the route. The methodology can be extended to evaluate the spatial-temporal performance by considering multiple routes.

The operational performance of the bus is estimated using (AFC) data - Go-card, Translink. For this pilot study, AFC data for 19\textsuperscript{th} August 2013 is used. Other relevant data such as route length, section length between stops, schedule time table were obtained from the Translink website (http://translink.com.au).

5.1. Study route and data

Bus route 111 is one of the major bus routes in Brisbane with high passenger demand. It connects the south (Eight Mile Plains) with the Brisbane CBD (see Figure 3) along a continuous Bus Rapid Transit corridor. With regard to the inbound direction (toward CBD), there are a total of 11 bus stops along the route, commencing at Eight Mile Plains Busway Station, and terminating at Roma Street Busway Station. The total length of the route is 17 km, and the average schedule travel time is 27 minutes.

Figure 3: Bus route 111 map (source: Google maps)
AFC data from Translink includes details of the individual passenger journey from smart card. This includes the following fields: *operator*, *operation date* (date corresponding to the bus operation), *smart card ID* (encrypted smart card ID of the passenger), *route* (bus route used by the passenger), *direction* (inbound or outbound), *schedule start* (the schedule start time of corresponding trip), *actual start* (the actual departure time of bus at the starting stop), *actual end* (the actual arrival time of bus at the destination), *boarding and alighting stop* (the stop ID that passenger uses to board and alight), *boarding and alighting time* (the time that passenger touch on or touch off the smart card when boarding or alighting), *vehicle ID* (encrypted ID of bus vehicle), *journey ID* (encrypted ID of bus trip), and *ticket type* (the type of smart card used by passenger such as adult, student or school children).

The template of smart-card data is expressed in Table 2, which shows that smart-card data can provide information to reconstruct vehicle’s service performed along all consecutive segments composing a transit route during a given time window (a day or an hour).

**Table 2. Template of smart-card data in Brisbane, Australia**

<table>
<thead>
<tr>
<th>Operator</th>
<th>Operations Date</th>
<th>Smartcard ID</th>
<th>Route</th>
<th>Service</th>
<th>Direction</th>
<th>Scheduled Start</th>
<th>Actual Start</th>
<th>Actual End</th>
<th>Boarding stop</th>
<th>Alighting stop</th>
<th>Boarding time</th>
<th>Alighting time</th>
<th>Vehicle ID</th>
<th>Journey ID</th>
<th>Ticket type</th>
</tr>
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<tbody>
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<td></td>
</tr>
<tr>
<td>Actual End</td>
<td>Boarding stop</td>
<td>Alighting stop</td>
<td>Boarding time</td>
<td>Alighting time</td>
<td>Vehicle ID</td>
<td>Journey ID</td>
<td>Ticket type</td>
<td></td>
<td></td>
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</table>

Steps implemented to extract needed inputs and outputs from smart-card data are shown in Figure 4 where inputs and outputs are extracted utilising the aforementioned smart-card data fields:
1. Based on the raw smart-card data, data for a given route and direction (inbound and outbound) is separated.

2. Based on the working day calendar and month index, data for a given month and working day only are extracted.

3. Based on the day index, data for a given working day are extracted. Data for a given vehicle then will be extracted on the basic of vehicle index.

4. Based on the schedule starting time index, data for each service of a given vehicle are extracted.

5. Service data for a given segment of bus route are extracted on the basic of alighting stop index and boarding stop index. Transit work then can be calculated for each service based on segment data (see Equation 1 in section 3).

6. Based on the actual starting time \( t_0 \) and actual ending time \( t_1 \) index of each service, the actual travel time \( \Delta t \) of a given service is calculated as follows: \( \Delta t = t_1 - t_0 \). Comparing the arrival time of bus vehicle at bus stops and ending point with schedule time can compute the on-time performance (OTP) indicator. OTP is defined as the proportion of observed trips that arrives the stops and ending point of the trip on time, where “on time” is less than 1 minute early and less than 5 minutes late.

7. The total number of passenger equals to the total number of boarding passenger or alighting passenger. At each bus stop, smart-card data can provide the first and last alighting time as well as the first and last boarding time, if there are passengers boarding and alighting. Thus, it enables to determine a proxy dwell time (2013), and the time that a bus vehicle arrives at a given stop.

Based on the aforementioned steps, performance indicators (OTP, Transit work, and Total travel time) of route 111 with inbound direction have been extracted from the raw smart-card data for every hour of 19th Aug, 2013. The operation of route 111 during one hour is regarded as a DMU in the DEA model.

Table 3 shows the briefly statistical description of the inputs (Number of services and Total travel time) and outputs (OTP and Transit work). Table 4 expresses the major performance indicators extracted for 111 where the Hour starts from 6 because there is no bus service from 0:00 to 5:00 am. OTP is defined in the current paper as the proportion of observed trips that arrive the ending point of the trip on time (not account for arrival time at stops). In some hours (such as from 22:00 to 24:00), there are not any services that arrive the destination on time, so OTP is 0.

Table 3. Statistical description of the inputs and outputs

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of services</td>
<td>4</td>
<td>1</td>
<td>11</td>
<td>2.74</td>
</tr>
<tr>
<td>Total travel time (hour)</td>
<td>1.73</td>
<td>0.32</td>
<td>6.52</td>
<td>1.54</td>
</tr>
<tr>
<td>Average travel time (hour)</td>
<td>0.43</td>
<td>0.32</td>
<td>0.59</td>
<td>0.06</td>
</tr>
<tr>
<td>OTP (%)</td>
<td>25</td>
<td>0</td>
<td>82</td>
<td>22.99</td>
</tr>
<tr>
<td>Transit work (p-km)</td>
<td>1193</td>
<td>46</td>
<td>7832</td>
<td>1975</td>
</tr>
</tbody>
</table>

5.2. Data analysis and results

This section investigates the service effectiveness of one bus route during every hour of a working day on the basis of maximizing the outputs. DMU thus is defined as the performance of bus route 111 in an hour (all bus services in an hour). However, due to the
duration of one bus service can across the two different hours, bus services in a given hour are selected based on the schedule start time. CCR model with output orientation is used to calculate the efficiency score of DMUs. The *service effectiveness* relates the service output offered by the operators to the service consumption (refer Figure 1), the benchmarking for which should help to maximize the bus ridership and quality of service. The rationale behind the selection of input and output is as follows:

**Input variables**: the variables should represent service outputs offered by the operator. Here, we select *number of service and total travel time*. The *number of service* provided in an hour represents the bus capacity offered by the operators. *Total travel time* is the sum of travel time of all services in a given hour. This study considers *total travel time* as input because we are focusing on the service effectiveness for which *total travel time* represents the duration for which the service is offered.

**Output variables**: the variables should represent the service consumption. Here, we select *Transit work* and OTP. *Transit work* by definition represents the service consumption of the community. Note: OTP is generally used as a variable of service output (Sheth et al., 2007). We argue that the transit operators in principle desire to maximize the OTP to increase the transit quality of service. OTP is used as an output by Barnum *et al.* (2008) to measure the performance of multiple bus routes. Therefore, we consider OTP as the second output in this paper.

Table 4: Bus route 111’s performance indicators for inbound direction (19th Aug, 2013)

<table>
<thead>
<tr>
<th>Hour</th>
<th>No of services</th>
<th>Transit work (p-km)</th>
<th>Total passenger (p)</th>
<th>Total travel time (hour)</th>
<th>Average travel time (hour)</th>
<th>OTP (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>4</td>
<td>1600</td>
<td>137</td>
<td>1.65</td>
<td>0.41</td>
<td>25</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>3605</td>
<td>300</td>
<td>2.50</td>
<td>0.42</td>
<td>17</td>
</tr>
<tr>
<td>8</td>
<td>11</td>
<td>7832</td>
<td>738</td>
<td>5.17</td>
<td>0.47</td>
<td>9</td>
</tr>
<tr>
<td>9</td>
<td>11</td>
<td>5377</td>
<td>563</td>
<td>6.52</td>
<td>0.59</td>
<td>82</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>1468</td>
<td>141</td>
<td>1.43</td>
<td>0.48</td>
<td>67</td>
</tr>
<tr>
<td>11</td>
<td>4</td>
<td>1193</td>
<td>115</td>
<td>1.77</td>
<td>0.44</td>
<td>25</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>545</td>
<td>63</td>
<td>1.28</td>
<td>0.43</td>
<td>33</td>
</tr>
<tr>
<td>13</td>
<td>3</td>
<td>1155</td>
<td>121</td>
<td>1.20</td>
<td>0.40</td>
<td>0</td>
</tr>
<tr>
<td>14</td>
<td>4</td>
<td>1257</td>
<td>136</td>
<td>1.73</td>
<td>0.43</td>
<td>25</td>
</tr>
<tr>
<td>15</td>
<td>4</td>
<td>1082</td>
<td>125</td>
<td>1.77</td>
<td>0.44</td>
<td>50</td>
</tr>
<tr>
<td>16</td>
<td>5</td>
<td>1508</td>
<td>197</td>
<td>2.20</td>
<td>0.44</td>
<td>40</td>
</tr>
<tr>
<td>17</td>
<td>6</td>
<td>1501</td>
<td>188</td>
<td>2.52</td>
<td>0.42</td>
<td>33</td>
</tr>
<tr>
<td>18</td>
<td>6</td>
<td>1296</td>
<td>158</td>
<td>3.22</td>
<td>0.54</td>
<td>33</td>
</tr>
<tr>
<td>19</td>
<td>4</td>
<td>508</td>
<td>57</td>
<td>1.75</td>
<td>0.44</td>
<td>25</td>
</tr>
<tr>
<td>20</td>
<td>2</td>
<td>380</td>
<td>42</td>
<td>0.77</td>
<td>0.38</td>
<td>0</td>
</tr>
<tr>
<td>21</td>
<td>3</td>
<td>174</td>
<td>22</td>
<td>1.10</td>
<td>0.37</td>
<td>0</td>
</tr>
<tr>
<td>22</td>
<td>3</td>
<td>259</td>
<td>35</td>
<td>1.18</td>
<td>0.39</td>
<td>33</td>
</tr>
<tr>
<td>23</td>
<td>1</td>
<td>46</td>
<td>5</td>
<td>0.32</td>
<td>0.32</td>
<td>0</td>
</tr>
<tr>
<td>24</td>
<td>1</td>
<td>118</td>
<td>8</td>
<td>0.33</td>
<td>0.33</td>
<td>0</td>
</tr>
</tbody>
</table>
To demonstrate the influence of variables to the DEA efficiency scores of DMUs, this paper computes the DEA efficiency scores for three cases, in which each case has a different combination between input and output variables (see Table 5). The results obtained from the efficiency analysis of the three cases are expressed in Figure 5. Here, case 1 (with one input and one output) illustrates the direct relationship between bus capacity and actual bus loading; case 2 considers the influence of travel time on the efficiency score of DMUs; and case 3 takes travel time as the second input and the OTP as the second output into account. The score axis illustrates the efficiency scores of DMUs (hourly operation of bus route). A DMU is efficient/effective if its score equals to 1, whereas lower score indicates that it is more inefficient/ineffective. For instance, hour 8 in case 1 is efficient (score equals to 1) and become benchmark for other inefficient DMUs (score < 1) whereas hour 6 with score of 0.56 is inefficient against hour 8. It is possible to increase the output of hour 6 by 78.6% (= (1-0.56)/0.56) using the similar inputs.

Table 5: Inputs and outputs using for DEA models in cases 1, 2, 3

<table>
<thead>
<tr>
<th>Case</th>
<th>DEA model</th>
<th>Orientation</th>
<th>Input variables</th>
<th>Output variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CCR (CRS)</td>
<td>output</td>
<td>Number of service</td>
<td>Transit work</td>
</tr>
<tr>
<td>2</td>
<td>CCR (CRS)</td>
<td>output</td>
<td>No of service, Total travel time</td>
<td>Transit work</td>
</tr>
<tr>
<td>3</td>
<td>CCR (CRS)</td>
<td>output</td>
<td>No of service, Total travel time</td>
<td>Transit work, OTP</td>
</tr>
</tbody>
</table>

Figure 5: The DEA efficiency score of the case 1, 2, 3

In case 1 and 2, there is only one efficient DMU (from 7:00 am to 8:00 am) which is the morning peak hour with the highest passenger demand. However, case 2 witnesses the slight increase of efficiency scores of DMUs in the afternoon compared to case 1 because they experience the lower travel time. Case 3 shows a significant increase of efficiency scores of most DMUs with two efficient DMUs at hours 8 and 10. It also expresses the significant growth of efficiency scores at hour 10 and 22 because at these two hours the OTP values are notably higher than the average value of the sample (25%). Those results are evident to state that OTP significantly influences the DEA efficiency scores, and the DEA
efficiency scores of inefficient DMUs are relative to the best performing DMUs (hour 8, 10). The efficiency scores can be utilised to identify the services which are:

- Best performance (benchmarks) with score 1,
- Good performance (score 0.8-1),
- Fairly good performance (score 0.5-0.8),
- Fairly bad performance (score 0.3-0.5), and
- Bad performance (score 0-0.3).

This work is substantially worth, especially identifying the benchmarks, because transit operator may find factors that lead to the inefficiency by looking at the best and the least performance hours. The study is currently extended to identify the reasons for the inefficient DMUs using bootstrap model (Simar & Wilson, 2007) in the second stage analysis.

6. Conclusion

Evaluating the temporal performance of individual transit routes within a transit system can help transit agencies to get insight into the operation of a transit route, and then identify the benchmarks and the factors that may result in the inefficiency of transit routes.

In this study we have reviewed the application of DEA models to measuring the transit performance at both system and route levels. Limited research is on the transit route performance evaluation. In literature, generally total passenger-km is used as the service consumption. We argue that transit work is a better service consumption indicator than passenger-km because the former incorporates the actual route length traversed by the passengers.

This pilot study has evaluated temporal performance of a single bus route. For this, CCR model is applied, with the required data obtained from the AFC database. The scores quantify the service effectiveness of the DMUs. The DMUs with low score should be further studied in the second stage analysis using truncated regression models to identify the reasons for the ineffectiveness. The knowledge gained will help to provide transit operators with additional information for decision makings.

This study indicates the significant contribution of OTP to the overall efficiency scores of DMUs. However, in the current analysis OTP is estimated based on the arrival time at the destination stop. Future research needs to use the arrival time at intermediate stops to enhance the accuracy of OTP and could use this method to compare the performance of different transit routes with different features such as the schedule time, the length of route and segments, and the size of vehicle.

7. Acknowledgements

The authors wish to thank TransLink Division of the Queensland Department of Transport and Main Roads, which has supplied the smart-card data of the transit system in Brisbane area.

References


