Motorway Travel Time Estimation: A hybrid model, considering increased detector spacing

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Abstract

Travel time is one of the indicators to quantify congestion and is an important parameter for the development and evaluation of strategies to mitigate congestion and its detrimental environmental and social impacts.

Travel time estimation and prediction on motorways has long been a topic of research. Prediction modelling generally assumes that the estimation is perfect. If the estimation is garbage, then no matter how good is the prediction modelling, prediction will also be garbage. Models have been proposed to estimate travel time from loop detector data. Generally, detectors are closely spaced (say 500 m) and travel time can be estimated accurately. However, detectors are not always perfect, and even during normal running conditions few detectors are not working, resulting in increase in the spacing between the functional detectors. Under such conditions, error in the travel time estimation is significantly large and generally unacceptable.

This research evaluates the in-practice travel time estimation model under different scenarios. Potential sources of errors (such as detector error, congestion build-up and dissipation, detector location) are identified. Thereafter, a hybrid model that can be easily adopted by motorway operators for time travel time estimation with acceptable accuracy limits is proposed and tested using simulation.

1. Introduction

Travel time is the time needed to travel between two points (say A and B) on the road network through a specific route consisting of multiple road sections/links. Individual vehicle travel time is the travel time (tt) of a vehicle i traversing from A to B along the respective route. Say n vehicles observed at A, between time-interval [t₁, t₂], are travelling to B along the same route. The average travel time for all the (n) vehicles is defined as follows (1)

$$TT = \frac{\sum_{i=1}^{n} t_{i}}{n}$$

Generally, in literature “travel time” refers to the average link/route travel time for all the vehicles. In this paper we also define travel time as the average travel time, and should not be confused with individual vehicle travel time.

Travel time “estimation” is the modelling of the observed traffic parameters (speeds and flows) into expected experienced travel time values. For instance, transforming flow and occupancy obtained from loop detectors at a specific location into expected experienced travel time on the road section. Travel time estimation is the basic requirement for travel time prediction, which assumes accurate estimation. It is an important network performance indicator, which quantifies the objectives of the planning, management and control policies.
Loop detector is the oldest and most widely available traffic data source. The detector can be signal loop or dual loop. Single loop detector generally provides vehicle counts and occupancy aggregated over a time interval, termed as detector detection interval (say 1 minute). Models are proposed to transform flow and occupancy into average space mean speed at the detector location [1-3]. Conceptually the relationship between detector data and spot speed can be expressed as follows:

\[ \bar{v}_i = \frac{N(i)}{O(i) \times g} \]  

(3)

Where: 
- \( \bar{v}_i \) = Space mean speed
- \( N \) = Vehicle counts during \( i^{th} \) time interval. It is also termed as ‘vehicle volume’ (vehicles/ time interval)
- \( O \) = Occupancy (proportion of time vehicles are over the detector over the time interval)
- \( g \) = Speed correction factor (inverse of mean effective vehicle length)

Generally, road authority assumes site-specific value for the speed correction factor based on assumed vehicle length, traffic conditions and detector configuration (Travel Time Data Collection Handbook, 1998). Wang and Nihan [4] have empirically proposed the following relationship between effective vehicle length and \( g \).

\[ g(i) = \frac{52.8}{\bar{L}(i)} \]  

(4)

Where: \( \bar{L}(i) \) = the mean effective vehicle length (feet) for the time interval \( (i) \). Effective vehicle length is the sum of the vehicle length and detectable length of loop detector.

Equation (3) can provide a good estimate of space mean speed at the detector location, if \( g \) is properly calibrated and traffic conditions are normal. Studies [3, 5, 6] have demonstrated that effective vehicle length is not a constant value, as is \( g \). Effective vehicle length depends on the distribution of different vehicle types in the stream which can significantly vary at different locations on the network and at different times of the day. Hence, it should be calibrated for location and time.

Dual loop detector also captures vehicle speed and classification in addition to the data provided by a single loop. Hence, the aforementioned issue related to the calibration of the \( g \) is not applicable to the data from dual loop detector.

For travel time estimation, the speed measured (from dual loop) or estimated (from single loop) at the detector location is to be generalised over the motorway section. Models [7-9] have been proposed where the speed between two consecutive detector locations are interpolated under different assumptions. For instance:

a) The Piecewise Constant Speeds Based (PCSB) method or Half distance method assumes that the point speed measured at the upstream and downstream detectors is constant for the first half and second half of the distance between the upstream and downstream detectors, respectively.

b) The Piecewise Linear Speed Based method (PLSB) [8] assumes that speeds vary linearly with time as a function of the distance between upstream and downstream detectors and hence are continuous between section (link) boundaries, compared to the PCSB method where the speeds are discontinuous at the section boundaries.

The aforementioned generalisation should be sensitive to the detector spacing and traffic conditions over the section. Generally, on motorways detectors are closely spaced on
average around 500 m. However, detectors are not always perfect and even under normal conditions few detectors malfunction. This increases the effective spacing between the functional detectors. For instance Figure 1 illustrates spacing between detectors on Pacific motorway, Brisbane. Blue line (with dot points), is the actual spacing based on the infrastructure. Average spacing is around 600 m. However on a randomly selected morning peak period, few detectors malfunction, resulting in effective spacing between functional detectors exceeding 2000 m (see red line with triangle points).

![Figure 1: Spacing between detectors on Pacific motorway, Brisbane Australia. (Blue line: actual spacing based on infrastructure; Black line: Effective spacing between functional detectors during a randomly selected morning peak period).](image)

The objectives of this paper are:

a) To evaluate the in-practice motorway travel time estimation models during conditions when the aforementioned spatial generalisation of speed might fail; and

b) Propose a hybrid model aiming to address the issues identified above.

The paper is structured as follows: first the in-practice travel time estimation models are briefly described and are evaluated. Thereafter, the proposed hybrid is explained and testing using simulation. Finally the limitations of the model are discussed and paper is concluded.

### 2. Evaluating in-practice travel time estimation models

#### 2.1 PCSB and PLSB model explanation

Say, we define a motorway section between two consecutive detectors (d, an d_i+1) at distance s apart. Average speeds \( V(d,t) \) from detector \( d \) at time \( t \) from the detector are discretely obtained at time intervals, DI seconds (say 60 seconds). Figure 2a and Figure 2b illustrates, the spatial generalisation of the speeds obtained from the detectors during detection interval at time \( t \) using PCSB (5) and PLSB (6) methods, respectively.

\[
v(x,t) = \begin{cases} 
    v(d_i,t) & \forall x \in (x_i, x_i + \frac{s}{2}) \text{ and } t \in [t,t-DI] \\
    v(d_{i+1},t) & \forall x \in (x_i + \frac{s}{2}, x_{i+1}) \text{ and } t \in [t,t-DI]
\end{cases} \tag{5}
\]

\[
v(x,t) = v(d_i,t) + \frac{v(d_{i+1},t)-v(d_i,t)}{s}(x-x_i) \quad \forall x \in [x_i, x_{i+1}] \text{ and } t \in [t,t-DI] \tag{6}
\]
Where: $x_i$ and $x_{i+1}$ are the distance coordinates for the loops $d_i$ and $d_{i+1}$, respectively in the time space region as shown in the figure.

Figure 2: Systematic representation of spatial generalisation of speed from a) PCSB and b) PLSB models

Once the speeds are generalised discretely, travel time from upstream to downstream can be defined by using trajectory method [8] where average trajectory of a vehicle, as discussed below, is traced considering the generalised speeds over space and time.

Here, the time space-region is divided into rectangular grids (of size $\Delta t$ by $\Delta x$ along time and space axis, respectively). The size of the grid is different for both PCSB and PLSB:

a) $\Delta t$ is DI or its multiple for both PCSB and PLSB
b) $\Delta x$ for PCSB is $s/2$, and $\Delta x$ for PLSB is defined by the user and can be set to 100 m.

The speed in each grid is considered as fixed and is discontinuous at the grid boundaries. From the start of each grid at upstream, imaginary vehicle trajectories (assumed to represent the average profile of vehicles in the grid) is traced under the assumption of constant speed in each grid. Average travel time for different time periods, is directly obtained from the trajectories.

2.2 Performance evaluation

Here, a microscopic motorway model (in AIMSUN [10]) is developed. Figure 3 illustrates the study section where we assume that congestion is spreading from downstream to upstream. We evaluate the performance of travel time estimation (over 2 km long section) under the following detector configurations:

a) Five detectors equally spaced at 500 m (detectors 1 to 5 in Figure 3)
b) Three detectors equally spaced at 1000 m (detectors 1, 3 and 5 in Figure 3)
c) Two detectors 2000 m apart (detectors 1 and 5 in Figure 3)

Different scenarios related to traffic congestion (free-flow, congestion build-up, congested and congestion dissipation) are simulated, and the performance of the travel time estimation models is evaluated independently for the aforementioned detector configurations. API's are written to extract the individual vehicle travel time and detector data. Detector data includes harmonic mean speed and flow aggregated over 1 minute interval. Simulated individual vehicle travel time is averaged over the 1 minute interval and is considered as ground truth.
Figure 3: Systematic representation of the study section, where congestion spreads from downstream to upstream.

Different traffic demand scenarios (s=1 to S) with replications (r= 1 to R) are simulated. The performance is evaluated in terms of the following indicators:

a) $A_{m}(8)$: This is the average of the accuracies ($A(s,r,n)$ (7)) obtained from all the estimation periods, replications and scenarios. It indicates the average performance, and is mathematically equivalent to 100(%) minus MAPE (Mean Absolute Percentage Error).

$$A(s,r,n) = 1 - \frac{|TT_{\text{Act}}(s,r,i) - TT_{\text{Est}}(s,r,i)|}{TT_{\text{Act}}(s,r,i)}$$

$$A_m(\%) = \frac{\sum_{s=1}^{S} \sum_{r=1}^{R} \sum_{n=1}^{n} A(s,r,n)}{S* R* n}$$

Where $TT_{\text{Act}}(s,r,i)$ and $TT_{\text{Est}}(s,r,i)$ are the actual average travel time (from simulated vehicle) and estimated (from model) travel time, respectively during $i^{th}$ estimation period (a minute each) from $r^{th}$ replication and $s^{th}$ scenario.

b) $A_s(9)$: This is the $5^{th}$ percentile of the individual accuracies obtained ($A(s,r,n)$ (7)) which means that 95% of the times the accuracy is more than $A_s$.

$$A_s(\%) = 5^{th} \text{percentile of } A(s,r,n)$$

Figure 4 presents typical results from one of the simulation runs. Figure 4a, Figure 4b and Figure 4c illustrates results from PCSB and PLSB when detector spacing is 500 m, 1000m and 2000 m, respectively. As expected, it is observed that during free-flow and fully congested conditions both PCSB and PLSB are close to the actual travel time and hence are not sensitive to detector spacing. This is because under such conditions, there is not significant variation of speeds along the section and speeds are well represented by both upstream and downstream detectors. However, for larger spacing (Figure 4c) both PCSB and PLSB does not perform well during congestion build-up and congestion dissipation periods.
Figure 4: Time series of travel time from PCSB and PLSB models compared with that of actual travel time for a) 500 m detector spacing; b) 1000 m detector spacing and c) 2000 m detector spacing

Figure 5 and Figure 6 presents the results of the overall performance (for different simulations and replications) of PCSB and PLSB models, respectively under congestion build-up and congestion dissipation for different detector spacing. Figure 5a and Figure 6a are for $A_m$ (%) representing the average performance. Figure 5b and Figure 6b are for $A_5$ (%) representing the reliability of the performance. Following is concluded from the figures:

a) The models provide good results (higher accuracy, and reliability) for lower detector spacing (500 m). Here, the generalisation is over 500 meters only, and not significant variation in speed between the two consecutive detectors is expected.

b) However, for higher detector spacing (2000 m) there is significant drop (over 10% compared to 500 m spacing) in $A_m$ and $A_5$ during congestion build-up and dissipation periods. This indicates that if the detector are closely spaced, then both the models provides good results, whereas if the detector spacing is large, then the models fails to provide good estimates during congestion build-up and dissipation periods.

c) Interestingly, simulation results show PCSB having slight better performance than PLSB, which means that linear generalisation not necessarily, provide better estimates. During congestion build-up and dissipation periods there can be significant variation of speeds obtained at two detector locations, especially if they are far apart. This variation is not necessarily linear. Figure 7 represents spatial variation of speeds on a 2000 m long section with bottleneck at downstream. Each sub plot has x-axis as space (detector location) from upstream to downstream; Y-axis as the speed. The sequence of plots represents the speed profiles at 1-minute time interval. These plots are obtained by placing detectors every 50 m on the 2000 m section. Here, initially all the detectors were at free-flow speed, during congestion build-up downstream speed has dropped to 40 km/hr that propagated upstream at shockwave speed. The upstream speed during the initial periods of the congestion build-up is close to free-flow and is maintained for certain space. For instance, plot at time=9 min since
the start of congestion, has free-flow speed for more than first half of the study section, whereas for other half, the speed drops linearly. Here the first half of the space is well represented by PCSB. Similarly, at time = 25 min, the downstream of the section (second half) is fully congested (speed = 40km/hr) and should be well represented by PCSB. These space-speed sub-plots indicate that the speed generalisation along the space for long section can be close to the PCSB than that of PLSB, though both having errors.

The above analysis indicates that both PCSB and PLSB provide good estimates when the section is either free-flow, fully congested or detector spacing is small. We utilise this finding to propose a hybrid model with an aim to improve the performance during congestion build-up and dissipation periods and higher detector spacing.

Figure 5 a) $A_m$ (%) and b) $A_5$ (%) for PCSB versus detector spacing. Y-axis is from 40% to 100%.

Figure 6 $A_m$ (%) and b) $A_5$ (%) for PLSB versus detector spacing. Y-axis is from 40% to 100%.
3. Proposed hybrid travel time estimation model

Models have strengths and limitations. Here we make a hypothesis, that different models can be integrated to enhance the performance of the hybrid model. The model proposed here, integrates cumulative plots based travel time estimation with PCSB model. Here, we choose PCSB because of its simplicity and comparable performance to that of PLSB.

3.1 Cumulative plot based travel time estimation

Cumulative plots are the time series of cumulative number of vehicles observed at a specific location. Say we have a loop detector at the upstream entrance and the downstream exit of the section. The counts from the detector can be cumulated to defined, \( U(t) \) and \( D(t) \) as two cumulative plots at upstream entrance and downstream exit, respectively. Theoretically, the average travel time for all the vehicles that arrives at upstream between time \( t_1 \) and \( t_2 \) is defined by equation (10) (see Figure 8). The assumption here is that the vehicles which corresponds to the cumulative counts at upstream during time \( t_1 \) and \( t_2 \) also corresponds to the cumulative counts at downstream during time \( t_3 \) (\( =D^{-1}(U(t_1)) \)) and \( t_4 \) (\( =D^{-1}(U(t_2)) \)).

\[
TT = \frac{\int_{t_1}^{t_2} D(t) - \int_{t_1}^{t_2} U(t)}{U(t_2) - U(t_1)}
\]

Where

\[
t_3 = D^{-1}(U(t_1)) \quad \text{and} \quad t_4 = D^{-1}(U(t_2))
\]
Figure 8: Graphical representation of classical cumulative plot technique for travel time estimation.

Cumulative plots are defined based on the detector counts. Detectors are not always perfect and even under normal running conditions one can easily observe 5% random error in the counting due to reasons such as, cross-talk, pulse break-up, closely spaced vehicles, hanging etc. The counting error in detector induces relative deviations (RD) between the plots (or drifts the plots). Moreover, presence of mid-section sink/sources such as off-ramp/on-ramp invalidates the aforementioned assumption and further adds to the RD. This model is vulnerable to the RD issue, which make the model practically not applicable. For instance in Figure 9, upstream detectors have counting error. U'(t) is the cumulative plot obtained from detector with errors. Comparing the area in Figure 9, with that of Figure 8, one can see that the total travel time is not the same.

Figure 9: An example to represent errors in travel time estimation when upstream detector is not perfect.

In order to address aforementioned relative deviation in cumulative plots, Bhaskar et al., [11] has proposed a model termed CUMulative plots and PRobe Integration for Travel time Estimation (CUPRITE), where cumulative plots [12] are integrated with probe vehicle data and RD issue is resolved. CUPRITE has been tested and validated on the signalised urban network for estimation of travel time statistics (average and quartiles) [13]. CUPRITE has been extended for successful exit-movement specific average travel time estimation on
signalised arterials [14]. In this paper, we adopt the framework of CUPRITE with the originality of using “virtual” probes defined by PCSB method and simplification of the CUPRITE application by defining “treatment periods”. We test the performance of the proposed model for travel time estimation under large detector spacing.

3.2 Proposed model architecture
The overall architecture for the proposed hybrid model is systematically illustrated in Figure 10. Time series of travel time $TT(t)$ using PCSB is estimated using the detector data (see Section 3.3). If the spacing between the functional detectors ($S$) is less than threshold ($S_{Th}=1000$ m) then the estimated $TT(t)$ is considered as good representation of the travel time on the study link. If not, then $TT(t)$ is to be corrected with the following steps:

- Define cumulative plots $U(t)$ and $D(t)$, at upstream entrance and downstream exit of the section
- Define treatment periods (see Section 3.5)
- For each treatment period:
  - Define virtual probes [VP] (see Section 3.6)
  - Fix [VP] to $D(t)$ (see Section 3.7)
  - Define point to pass [P2P] (see Section 3.8)
  - Redefine $U(t)$ (see Section 3.9)
  - Estimate travel time using redefined $U(t)$ and $D(t)$ (see Section 3.10)
  - Correct the $TT(t)$ during the treatment period

The details for each step are explained in the following subsections with the help of an example.

3.3 Estimate the time series of PCSB travel time
As discussed in Section 2.1, the PCSB model is applied to define individual vehicle trajectories starting at upstream at regular time-periods (equal to the detector detection interval). Travel time from these trajectories is obtained to define time series of travel time $TT(t)$ along the study section. For instance, Figure 11 illustrates a time series of travel time from PCSB, where $TT(t)$ is the travel time at time $t$.

3.4 Define $U(t)$ and $D(t)$
Cumulative plots $U(t)$ and $D(t)$ at the upstream entrance and downstream exit of the study section are defined by cumulating the vehicle counts from the upstream and downstream detectors. For instance, Figure 12 illustrates cumulative plots for the site for which travel time is defined in Figure 11 (illustration only).

3.5 Define treatment period
Say, $T_{Thr}$ is the expected travel time during non-congested conditions. This parameter is to be tuned for the site and can be assigned as 105% of free-flow travel time of the study corridor. We first identify, time when the estimated travel time from PCSB is higher than $T_{Thr}$. Say $TT(t) > TT_{Thr}$ for $t_1 \leq t \leq t_2$ (Refer to Figure 11), then we define the start ($t_s$) and end ($t_e$) of the treatment period as follows

$$t_s = t_1 - b_1$$

$$t_e = t_2 + b_2$$

Where: $b_1$ and $b_2$ are the buffer time needed to account for the lag in detecting at the start of congestion build-up and at the end of congestion dissipation from PCSB model. We can consider $b_1 = b_2 = 30$ minutes.

Remaining steps are treated independently for each treatment period.
3.6 Define virtual probes for each treatment period

Here, for each treatment period, three virtual probes are defined in terms of the time when they are observed at upstream ($t_u$) and downstream ($t_d$) locations. Two of them are from the points in the PCSB travel time plots corresponding to the start and end of the treatment period and third one is the point corresponding to the maximum PCSB travel time within the treatment period. For instance, in Figure 11, time $t_s$ and $t_e$ corresponds to time at the start and end of treatment period, respectively and time $t_m$ is the time corresponding to the maximum PCSB travel time within time $[t_s, t_e]$. The maximum PCSB travel time should correspond to the condition when the section is most congested and should be the most accurate point of PCSB within the current treatment period.

Refer to first three columns of Table 1: The first column is the virtual probe coordinates in the PCSB time series plot. The second and third columns are the time when the virtual probe is observed at upstream ($t_u$) and downstream ($t_d$) location, respectively. Here, the third column is defined by adding respective travel time (from first column) to the time $t_u$ (from second column).
Figure 10: Architecture for the proposed hybrid model
Figure 11: Systematic illustration of the treatment period and virtual probes.

Figure 12: Systematic illustration of the U(t) and D(t) with virtual probes fixed to D(t), and portion of the U(t) to be redefined identified.
3.7 Fix virtual probe to the downstream cumulative plot
The virtual probes information is only the time when they are observed at the upstream and downstream location. In order to integrate them in cumulative plots, we fix them to the downstream cumulative plot, i.e., define their rank (cumulative count value) considering D(t). Refer to the fourth column of Table 1, where the rank for each virtual probe is defined by plugging \( t_d \) (third column) in the downstream cumulative plot function (\( D(t) \)).

3.8 Define point to pass
Once the virtual probes are fixed to \( D(t) \) we define the points from where \( U(t) \) should pass so as to be consistent with the travel time from virtual probes. The points to pass from the three virtual probes are:

a) \( P_s = (t_s, R_s) \);

b) \( P_m = (t_m, R_m) \); and

c) \( P_e = (t_e, R_e) \)

Where: \( R_s \), \( R_m \) and \( R_e \) are ranks as defined in the previous section.

Table 1: Defining virtual probes for each treatment period and their ranks in the cumulative plots

<table>
<thead>
<tr>
<th>Virtual Probe at PCSB time series plot</th>
<th>( t_u )</th>
<th>( t_d )</th>
<th>Rank in cumulative plots</th>
</tr>
</thead>
<tbody>
<tr>
<td>((t_s, TT(t_s)))</td>
<td>( t_s )</td>
<td>( t_s + TT(t_s) )</td>
<td>( R_s = D(t_s + TT(t_s)) )</td>
</tr>
<tr>
<td>((t_m, TT(t_m)))</td>
<td>( t_m )</td>
<td>( t_m + TT(t_m) )</td>
<td>( R_m = D(t_m + TT(t_m)) )</td>
</tr>
<tr>
<td>((t_e, TT(t_e)))</td>
<td>( t_e )</td>
<td>( t_e + TT(t_e) )</td>
<td>( R_e = D(t_e + TT(t_e)) )</td>
</tr>
</tbody>
</table>

3.9 Redefine upstream cumulative plot
Once the aforementioned three points to pass are defined, \( U(t) \) should be redefined considering each point to pass in order. For this only the portion of the upstream cumulative plot \((U(t))\) which is within the respective treatment period is considered (13).

\[
U'(t) = U(t) \quad \forall \ t \in [t_s, t_e]
\] (13)

Due to the errors in the detector counting and non conservation of vehicles along the study corridor, \( U'(t) \) may not pass through the points to pass. As these errors only affects the cumulative counts hence the adjustment in the cumulative plot should be along the cumulative count axis (vertical axis). For this, below mentioned vertical scaling and shifting technique is applied within the respective time.
Vertical shifting for $P_s$

The relative deviation ($\varepsilon$) in the $U'(t)$ at time $t_s$ is given by the difference between $P_s$ and $U'(t_s)$, hence this deviation is reduced by vertical shifting $U'(t)$ by the same amount and revised upstream plot is termed as $U_1(t)$. Refer to Figure 14a) for the self-explanatory illustration of vertical shifting for $P_s$.

\[ \varepsilon = U'(t_s) - R_s \]  

\[ U_1(t) = U'(t) - \varepsilon \]  

$R_s = D(t_s + TT(t_s))$

$R_m = D(t_m + TT(t_m))$

$R_e = D(t_e + TT(t_e))$
Figure 14 Systematic illustration of the redefining $U(t)$ a) Vertical shifting for $P_s$; b) Vertical scaling and vertical shifting for $P_m$; and c) Vertical scaling for $P_e$.
Vertical scaling and shifting for $P_m$
Considering $P_m$, we apply corrections on $U_1(t)$ (15) so that it passes through $P_m$. Here the relative deviation in $U_1(t)$ at time $t$ is defined as $\varepsilon_1(t)$.

For time $t$ ($t_s \leq t \leq t_m$): $\varepsilon_1(t)$ is the result of the accumulation of the relative deviation from $t_s$.

The relative deviation at time $t_s$ is zero, and at time $t_m$ is difference between $U_1(t_m)$ and $R_m$.

$$\varepsilon_1(t_s) = 0; \quad \varepsilon_1(t_m) = U_1(t_m) - R_m$$

(16)

We assume that the ratio of the relative deviation at time $t$ ($t_s \leq t \leq t_m$) to the cumulative counts since time $t_s$ to be constant (17).

$$\frac{\varepsilon_1(t)}{U_1(t) - R_s} = \frac{\varepsilon_1(t_m)}{U_1(t_m) - R_s} \quad \forall \ t \in [t_s, t_m]$$

(17)

For time $t > t_m$: $U_2(t)$ should be vertically shifted by $\varepsilon_1(t_m)$, so as to be continuous and monotonically increasing.

Rearranging the above equation (17), $\varepsilon_1(t)$, and $U_2(t)$ are defined as follows:

$$\varepsilon_1(t) = \begin{cases} 
(U_1(t) - R_s) \ast \left( \frac{U_1(t_m) - R_m}{U_1(t_m) - R_s} \right) & \forall \ t \in [t_s, t_m] \\
U_1(t_m) - R_m & \forall \ t > t_m 
\end{cases}$$

(18)

$$U_2(t) = U_1(t) - \varepsilon_1(t)$$

(19)

Vertical scaling for $P_e$
Similar to the previous step, considering $P_e$, we apply corrections on $U_2(t)$ (19) so that it passes through $P_e$. Here, we define $\varepsilon_2(t)$ as the relative deviation in $U_2(t)$ for time $t$.

For time $t \leq t_m$: As $U_2(t)$ was defined by considering $P_m$, $\varepsilon_2(t)$ for time $t \leq t_m$ should be zero.

For time $t$ ($t_m < t \leq t_e$): $\varepsilon_2(t)$ is the result of the accumulation of the relative deviation from $t_m$.

The relative deviation at time $t_m$ is zero, and at time $t_e$ is difference between $U_2(t_e)$ and $R_e$.

$$\varepsilon_2(t_m) = 0; \quad \varepsilon_2(t_e) = U_2(t_e) - R_e$$

(20)

Assuming the ratio of the relative deviation at time $t$ ($t_m \leq t \leq t_e$) to the cumulative counts since time $t_m$ to be constant (21).

$$\frac{\varepsilon_2(t)}{U_2(t) - R_m} = \frac{\varepsilon_2(t_e)}{U_2(t_e) - R_m} \quad \forall \ t \in (t_m, t_e]$$

(21)

Rearranging the above equation (21), $\varepsilon_2(t)$, and $U_3(t)$ are defined as follows:

$$\varepsilon_2(t) = \begin{cases} 
0 & \forall \ t \leq t_m \\
(U_2(t) - R_m) \ast \left( \frac{U_2(t_e) - R_e}{U_2(t_e) - R_m} \right) & \forall \ t \in (t_m, t_e] 
\end{cases}$$

(22)

$$U_3(t) = U_2(t) - \varepsilon_2(t)$$

(23)
3.10 **Estimate average travel time**

Refer to Figure 15 the average travel at time \( t \) \( (t_s \leq t \leq t_e) \) during estimation period of \( \Delta t \) (say 1 minute) is defined considering \( U_3(t) \) and \( D(t) \) as follows, which is the discrete equation for estimating the average area under the cumulative plots.

\[
\overline{TT}(t) = \frac{\sum_{t=D^{-1}(U_3(t)+\Delta t)}^{t-D^{-1}(U_3(t))} D(t) - \sum_{t=t}^{t+\Delta t} U_3(t)}{U_3(t+\Delta t) - U_3(t)}
\]  

(24)

![Figure 15: Average travel time estimation utilising D(t) and U_3(t)](image)

The above process is repeated for different treatment periods and the time series of travel time during the treatment periods is corrected by the travel time obtained from the hybrid model during different treatment periods.

The proposed model is tested using simulation on the simulation model defined in section 2.2. Different scenarios considering 10% random error in the detector counting is considered for the spacing of 2000 meters between the detectors. The performance is separately testing during congestion build-up and congestion dissipation-periods and reported in terms of \( A_m(\%) \) (see Figure 16a) and \( A_5(\%) \) (see Figure 16a) and compared to PCSB model. The results are very motivating, not only the mean accuracy but also the reliability of the travel time estimation is significantly increased. The results show around 10% improvement in accuracy and around 15% improvement in reliability \( (A_5(\%)) \).
Motorway Travel Time Estimation: A hybrid model, considering increased detector spacing

Figure 16 Evaluation of proposed model against the existing models a) $A_m(\%)$ b) $A_5(\%)$

4. Model Limitation

The proposed hybrid model identifies a treatment period, and considers the maximum travel time during the treatment period as the most accurate PCSB travel time point to be used for correcting the cumulative plots. The fusion of the models (cumulative plot and PCSB) should provide better performance than the independent model. However, if maximum travel time during the treatment period is not a good representative of the actual travel time during congested conditions then, there will be marginal improvement in the performance of hybrid model compared to that of PCSB. The scenario when such correction points cannot be identified by PCSB includes situations when the congestion occurs within the section and is not detected by the loops located at the upstream and downstream ends of the section.
5. Conclusion
It has been shown that existing models are applicable only on dense spacing of detectors (typically 500 m), which practically is not always achievable. The proposed hybrid model provides good estimates of travel time for wider detector spacing. This provides opportunities to consider options for reduction in the number of detectors on motorways—-which can provide considerable cost (installation and maintenance) savings to the motorways stakeholder. The application of the proposed hybrid model should improve the accuracy and reliability of the offline network performance evaluation and the development of historical database of experienced travel time. This database is vital for the success of any real time and predictive traveller information system. The historical and real time estimated travel time profiles are basic requirements for predictive travel time modelling.

In this paper, the proposed model is tested using simulated data. We are currently in the process of validating the model with real data, where ground truth of travel time is obtained from Bluetooth [15].

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