# Innovations in modelling time-of-day choice

## The Auckland Regional Transport Model (ART3, 2008)

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*Abstract*—**With road user charging and toll roads potentially on the agenda for transport strategies in Auckland, it is important that traffic forecasts reflect the impact of charges varying by time of day. Consequently, for the new regional transport model (ART3), an innovative design for a time of day choice model was developed. This model recognises both international findings on the way in which this topic should best be modelled and sensible limits to the effort involved in developing and applying such techniques.**

**One important innovation in the development of ART3 is that the procedure estimates road user charges and other costs experienced on the complete tour (rather than for isolated trips), thus considering the times of travel of both the outward and return trips. Another significant innovation is that the procedure has been overlaid on a conventionally estimated aggregate hierarchical distribution and mode choice model system. As such, neither the costs of estimating this type of model, nor the run times taken to apply it are significantly affected.**

**In principle, the procedure presented in this paper could be added to existing multimodal transport models without the need for re-estimation.**

## I. Introduction

Strategic transport models seek to represent the travel behaviour of a population across a transportation network. In doing so, such models allow for the assessment of alternative scenarios leading to informed and appropriate transport policy decisions.

In principle, the fundamental task of any strategic transport model is to forecast the transport *choices* made by the modelled population in the context of a future scenario. Such choices relate particularly to mode, destination, route or the subject choice of this paper; time-of-day (ToD).

It is well known that a population's behaviour with regard to the time of travel is not static. That is, individuals will alter their time of travel in response to changes in congestion levels.

Recent global interest has emerged with regard to policies that tackle congestion via road user charging. In many instances, such scenarios seek to influence travellers' ToD decisions by the application of differential pricing across the day. It is clear that strategic transport models used to assess such policies must represent the population's sensitivity to such cost changes with an appropriate level of fidelity. However, the great majority of currently implemented strategic transport models do not and thus cannot hope to model such scenarios with any degree of certainty.

In 2007, the Auckland Regional Council (ARC) commissioned Sinclair Knight Merz Ltd, in conjunction with Beca Infrastructure Ltd and David Simmonds Consultancy, to develop new regional land use and transport models; the latter known as the Auckland Regional Transport Model (ART3). It was a requirement that this transport model have the capability to assess road pricing strategies with potential variation in cost by ToD.

The underlying structure of ART3 is a conventionally estimated, 4-stage, aggregate hierarchical model (Ortúzar & Willumsen, 2001). In order to incorporate time-of-day choice into this model, it was necessary to re-arrange the model so that the representation of travel for ToD choice could operate as tours rather than trips.

The paper proceeds as follows; in the following section, a brief overview of the current state of practice with regard to ToD choice modelling is presented. Section III then provides an overview of the ART3 model, with particular focus on the ToD choice component. The choice modelling methodology is then formally presented in section IV. This is followed by section V; an overview of the procedure used to calibrate the ToD choice model. The paper concludes with a brief summary.

## II. BACKGROUND

There exists an abundance of literature regarding the implications of departure time choice for highway project modelling. Booz Allen Hamilton, 2003 provides an extensive review of this literature, suggesting that although some developments exist in the USA, the UK is further advanced. The UK Department for Transport (DfT) provides a web-based guidance for appraising multimodal transport projects and proposals known as WebTAG. This guidance states;

*"…Thus, as a core requirement, properly formulated variable demand and traffic assignment models are required to refine the preferred options and to support the business case. The variable demand model should include modules representing trip frequency, mode choice, macro time of day and trip distribution."*(WebTAG, unit 3.12.2, para 1.1.9)

In terms of how macro time period choice (between the broad modelled time periods) should be incorporated in models, the guidance further states that:

*"There is limited evidence on the strength of the macro time choice mechanism. Recent Departmental research suggests that time period choice is generally more sensitive to changes in travel conditions than mode choice."*
(WebTAG, unit 3.10.3, para 1.8.4)

Further discussion in this WebTAG unit makes clear that the likely position of macro time-of-day choice in most hierarchical models would be at a similar level to mode choice but higher than distribution, which is usually found to be more sensitive (WebTAG unit 3.10.3, paras 1.9.4, 1.11.15-17).

It has been demonstrated that if one wishes to use discrete choice theory for ToD choice modelling, the theoretically consistent way to proceed, given these hierarchical requirements, is to move to a tour-basis (Gordon et al., 2007). Using a tour-basis, the simultaneous choice of outbound and return time periods are modelled, conditional on the tour cost for that time period combination.

As the DfT guidance currently stands, there is some tension between the general recommendations for PA modelling and ToD choice modelling (though tour modelling is not ruled out). In other words, there is no recommendation to use a tour approach, but at the same time the hierarchical requirements for time of day modelling remain. In light of this, it is likely that most TIF models[1] will either ignore time of day modelling completely, or resort to their own devices (the appropriateness of which will depend strongly on the expertise of the model architects).

[1] This is an acronym for "Transport Improvement Fund", but in this context can be treated as synonymous with road pricing.

Two exceptions should be made to the above discussion, in the UK context. The first is the APRIL model, which was specifically developed in the 1990s to assess road pricing in London: this contains a tour-based time of day choice model, as well as income segmentation. More recently, the PRISM model, developed by RAND Europe for the West Midlands region, has implemented both effects in its TIF work. However, this model contains several features that are different from conventional UK models. Although substantial model documentation is available, it has not been widely examined.

Thus, time-of-day choice modelling is a relatively new topic for which there is a range of different methods being discussed, researched and used internationally but, as yet, no conclusively leading methodology. Nonetheless, the ART3 implementation of ToD choice modelling, which we describe in this paper, appears to lie within the range of current thinking. Most importantly, the implementation is developed within the constraints of a conventional, trip-based, aggregate four step model.

## III. Time-of-day Modelling in ART3

ART3 is a 4 stage, multimodal city transportation model linked to the land use model; ASP. An illustration of the base structure of this model (i.e. excluding the ToD choice element) is presented in figure 1. With reference to this figure, the trip end, mode choice and distribution sub-models were to be estimated on 24 hour data (trips and generalised costs), following which time period factors would be applied to create peak and interpeak matrices for assignment purposes. The challenge was to design a ToD choice procedure which did not change (or indeed complicate) the estimation of the model presented in figure 1. In doing so the risks regarding estimation, budget and delivery timeline would be minimised.

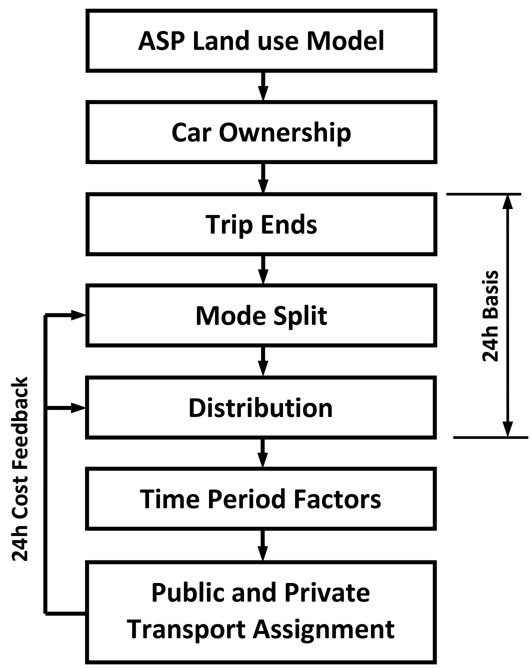


Figure 1. Base ART3 model structure

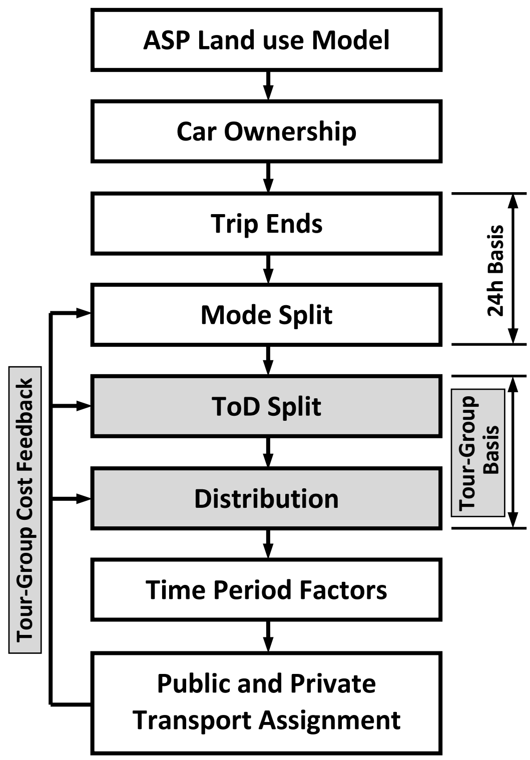


Figure 2. Modified ART3 model structure, incorporating ToD choice

The methodology developed for modelling ToD choice within the ART3 model involved introducing a ToD split within the destination and mode choice hierarchy, based on tour concepts (see figure 2). However, re-estimation of the model was not required.

Initial research demonstrated that the base model (i.e. figure 1), which had been estimated on a 24 hour trip basis, could be implemented on a tour basis and obtain consistent forecasts. With this new structure, the tours were classified into *tour groups* representing combinations of outbound and return time periods, and a ToD choice model was incorporated to predict changes in the proportions of tours in these groups as a result of, for example, congestion charging. In this way, while retaining consistency with the estimated 24 hour trip-based model, the implemented tour-based model became sensitive to time-specific transport strategies.

For ART3, five time periods were defined. Within the context of these time periods, four tour groups were defined (A-D in table 1). These groups were based on whether the outbound and return trips of the home-based tour were in either or both of the AM and PM peak periods. The base split of trips between the four tour groups for each mode and purpose combination was determined from the household travel survey, as presented in table 2.

The ToD split was implemented as an incremental logit model. That is, for the base scenario, the model splits tours into tour groups in accordance with the proportions given in table 2. For forecast scenarios, the time of day model modified the base split in accordance with changes in the relevant costs of travel by ToD. Section IV discusses this in more detail.

In summary, a methodology was developed by which ToD choice was represented in the ART3 model. This methodology was implemented in a manner consistent with discrete choice theory and enabled changes in the times of travel to be forecast.

TABLE I. ART3 TOUR GROUPS

| **Outbound Time Period** | **Return Time Period** | | | | |
|---|---|---|---|---|---|
| | ***AM Peak*** | ***Inter Peak*** | ***School Peak*** | ***PM Peak*** | ***Off Peak*** |
| ***AM Peak*** | B | B | B | A | B |
| ***Inter Peak*** | D | D | D | C | D |
| ***School Peak*** | D | D | D | C | D |
| ***PM Peak*** | D | D | D | C | D |
| ***Off Peak*** | D | D | D | C | D |

TABLE II. ART3 TOUR GROUP SPLITS BY MODE

| **Mode** | **Purpose** | **Tour Group** | | | |
|---|---|---|---|---|---|
| | | ***A*** | ***B*** | ***C*** | ***D*** |
| ***Car*** | ***Work*** | 39% | 21% | 15% | 25% |
| | ***Education*** | 14% | 59% | 5% | 22% |
| | ***Shop*** | 5% | 13% | 16% | 66% |
| | ***Other*** | 7% | 21% | 16% | 56% |
| ***Public Transport*** | ***Work*** | 53% | 23% | 14% | 10% |
| | ***Education*** | 19% | 65% | 6% | 11% |
| | ***Shop*** | 5% | 19% | 11% | 65% |
| | ***Other*** | 18% | 31% | 14% | 37% |

## IV. TIME-OF-DAY MODELLING METHODOLOGY

In this section, we formalise the ToD choice modelling methodology used for ART3. This methodology was based on two key innovations; (a) the concept of modelling tours rather than trips, and (b) the inclusion of a ToD split within the destination and mode choice model. These two innovations are detailed in the following two sub-sections. Although all examples presented are specific to the ART3 model, in the interests of generality model concepts are presented irrespective of the model specifics. For brevity and readability, in the main text we restrict discussion to an outline of the methodologies. Precise mathematical details are set out in the appendices.

### A. *Tour groups and associated costs*

The conventional basis for modelling person travel is the *trip*; each trip representing a single leg of the full home-to-home journey. However, as discussed in section II, when modelling ToD choice it is necessary to model travel behaviour on the basis of *tours* rather than trips. Each tour represents both the outbound (from-home) and return (to-home) trips made by a single traveller. For brevity, in this paper we address the modelling of home-based trips only, the modelling of non-home-based trips being trivial by comparison. Herein, outbound and return quantities are defined by the subscripts fh and th respectively.

To formalise our discussions regarding the modelling of temporal variation, we must first introduce our definitions of time segmentation. In the case of strategic transport models, typically discrete time periods are defined within which aggregate travel demand is modelled. In the case of the ART3 model, the following set of time periods were defined (essentially for the purposes of assignment):

$$t \triangleq \left\{\begin{array}{c}\text{AM-Peak (AM)}\\ \text{Inter-Peak (IP)}\\ \text{School-Peak (SP)}\\ \text{PM-Peak (PM)}\\ \text{Off-Peak (OP)}\end{array}\right\}. \tag{1}$$

A *tour group* is defined as a set of tours sharing common outbound and return time periods, denoted: $[\mathrm{t_{fh}}, \mathrm{t_{th}}]$. For example, the simplest such tour group contains tours with outbound and return trips occurring in the AM and PM periods respectively (i.e. [AM,PM]). For a model having $\tau$ time periods, $\tau^2$ potential tour groups exist. However, modelling all such groups separately is likely to be impractical. Consequently, aggregation is necessary to reduce the number of modelled tour groups to a manageable quantity. For the ART3 model, the set of modelled tour groups g contained the following four elements:

$$\begin{aligned}
g_1 &\triangleq \{[t_{\mathrm{fh}}, t_{\mathrm{th}}] | t_{\mathrm{fh}} = \mathrm{AM}, t_{\mathrm{th}} = \mathrm{PM}\},\\
g_2 &\triangleq \{[t_{\mathrm{fh}}, t_{\mathrm{th}}] | t_{\mathrm{fh}} = \mathrm{AM}, t_{\mathrm{th}} \neq \mathrm{PM}\},\\
g_3 &\triangleq \{[t_{\mathrm{fh}}, t_{\mathrm{th}}] | t_{\mathrm{fh}} \neq \mathrm{AM}, t_{\mathrm{th}} = \mathrm{PM}\},\\
g_4 &\triangleq \{[t_{\mathrm{fh}}, t_{\mathrm{th}}] | t_{\mathrm{fh}} \neq \mathrm{AM}, t_{\mathrm{th}} \neq \mathrm{PM}\}
\end{aligned} \tag{2}$$

Each of these tour groups were known to have significantly different characteristics, and experience significantly different costs of travel. Although possibly obvious, it is worth making the following remark; the information formally represented by the two variables trips and time-periods is now represented by the variables tours and tour-groups.

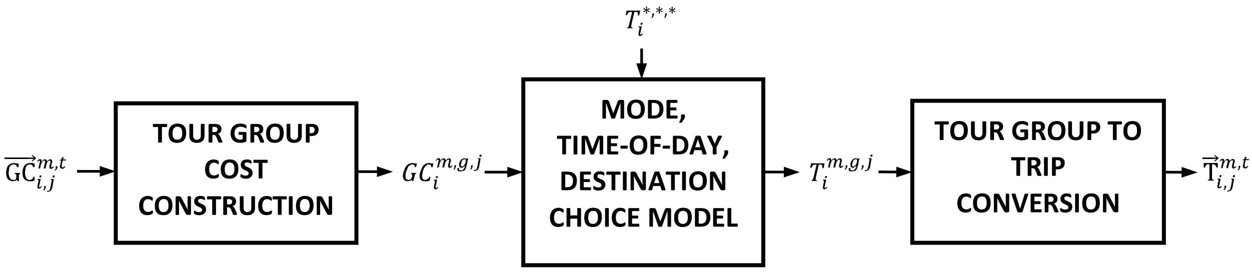


Figure 3. Top-level overview of methodology for modelling ToD travel choices

Figure 3 provides a top-level overview of the proposed methodology for modelling ToD travel choices. Central to this methodology is the mode, time-of-day and destination choice (MTD) model. It is this component of the model that seeks to emulate traveller decisions, splitting tours by production zone $i$: $T_i^{*,*,*}$ into tours by mode $m$, tour group $g$ and attraction zone $j$: $T_i^{mg,j}$. Driving these choices are a set of associated costs: $GC_i^{m,g,j}$; the generalised cost of travel experienced by mode $m$, tour group $g$ tours, produced in zone $i$ and attracted to zone $j$.A detailed discussion of this element of the model is deferred until the next section of this paper. At this point it is useful to explicitly define the subscripts: $m$, $g$, $t$, $i$ and $j$ as denoting mode, tour-group, time-period, origin/production and attraction/destination respectively.

It is necessary that the MTD model interface with the assignment element of the strategic transport model. In general, this element of the model operates on the basis of trips and rather than tours. Consequently, two mappings are required to allow for this interface. A discussion regarding these two mappings is the subject of the remaining text of this section.

To generate inputs to the MTD model, the generalised cost of travel for trips by origin-destination (OD) zone and time period: $\overrightarrow{\mathrm{GC}}_{i,j}^{m,t}$ is used to generate the cost of travel for tours by production-attraction (PA) zone and tour group: $GC_i^{m,g,j}$. Herein, the arrow accent: → is used to denote quantities with respect to an OD base, as distinct from quantities with either a PA base or no defined direction. In principle, this mapping is achieved as follows; as we have generalised costs by time period, and there is a correspondence between the time periods and tour groups, it is merely a matter of averaging the time period values (according to the distribution of trips) to obtain the appropriate value for the tour group. This averaging is conducted both temporally and spatially to represent geographic variation in ToD travel behaviour. Specific details of this process are contained in Appendix A1.

The second mapping necessary for the interface of the MTD model with trip assignment maps tours by mode, tour group and PA zone: $T_i^{m,g,j}$ to trips by mode, time period and OD zone: $\vec{\mathrm{T}}_{i,j}^{m,t}$. As with the mapping of generalised cost, this process is little more than a weighted averaging, the specific details of which are outlined in Appendix A2.

*B. Choice modelling*

The central choice modelling element of a conventional 4-stage strategic transport model is the distribution and mode split (DMS) model. It is this component of the model that seeks to represent the decisions travellers make with respect to destination and mode of travel. If one wishes to model the additional choice regarding time of travel, the DMS model is thus the natural component of the strategic model in which to impose this.

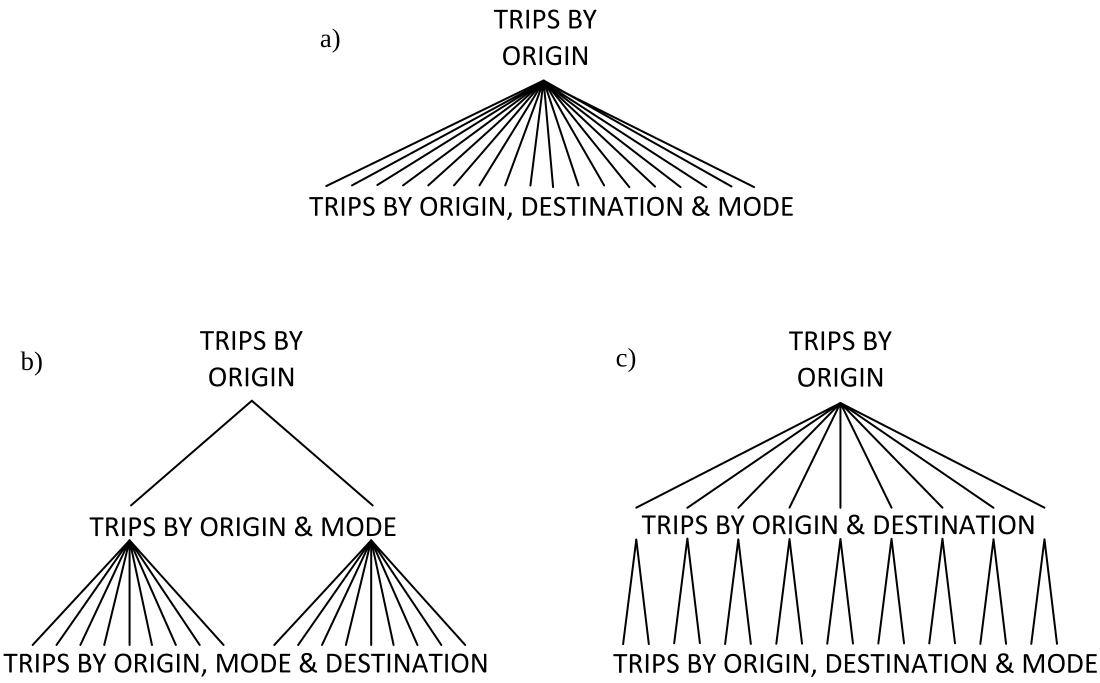


Figure 4. Conventional DMS models; a) simultaneous distribution and mode choice, b) pre-distribution mode choice, c) post-distribution mode choice.

The functional form of the conventional DMS model is that of either a nested or multinomial logit model. Three potential model hierarchies (configurations) exist, as depicted in figure 4. These configurations differ by the order in which trips are split by mode and destination. To be tractable with discrete choice theory, the sensitivity of the model must be lower at the upper level of the model hierarchy. For example, for the case of the pre-distribution mode choice DMS model (see figure 4(b)), the destination split is more sensitive to changes in cost than the mode split. In the case of a simultaneous model (see figure 4(a)), both splits are equally sensitive to cost changes. DMS models are typically calibrated to survey data using maximum likelihood techniques, the model hierarchy being determined by this calibration. It is a common misconception that the model hierarchy implies the order in which travellers make their decisions. However, this is not the case; nesting is simply used to relax the independence assumption required for the construction of the multinomial logit model (Greene, 2008).

Key to inserting a time-of-day split into the DMS model hierarchy is a judgement regarding the required ToD sensitivity of the resulting model relative to mode and destination (see section II). In this paper we present the specifics for the ‘insertion’ of a tour group split into a pre-distribution mode choice DMS model (i.e. figure 4(b)), however the principles demonstrated may be easily extended to apply to the other hierarchical structures.

Inserting a tour group split into a pre-distribution mode choice DMS model is of particular interest as this split is placed between the mode and destination splits. A pictorial summary of the resulting MTD model is presented in figure 5. For brevity, the remainder this section is dedicated to a top-level overview of this model, the precise mathematical workings being delayed until Appendix B1.

With reference to figure 5, the MTD model can be considered as being composed of two elements. The first of these performs the mode, tour group and destination splits that emulate traveller's decisions with regard to these alternatives (i.e. the RHS of figure 5). The second of these elements constructs the composite utilities (i.e. scaled costs) used to drive these splits (i.e. the LHS of figure 5). These two elements are linked by the attraction constraint. This constraint ensures trip attractions forecast by the MTD model agree with those forecast by the attraction model (a separate component of the 4-stage strategic model). The attraction constraint is imposed by a set of destination specific additive perturbations to the base utilities; $b_j$, the exact values of which are solved for via an iterative process as follows; at each iteration the attraction constraints are updated via a simple feedback law. This alters the base utilities, which in turn alters the composite utilities and the MTD splits in such a way that the error in the attraction constraint is reduced (see Appendix B1). As a consequence of the iterative procedure employed, exponential convergence of this error is guaranteed.

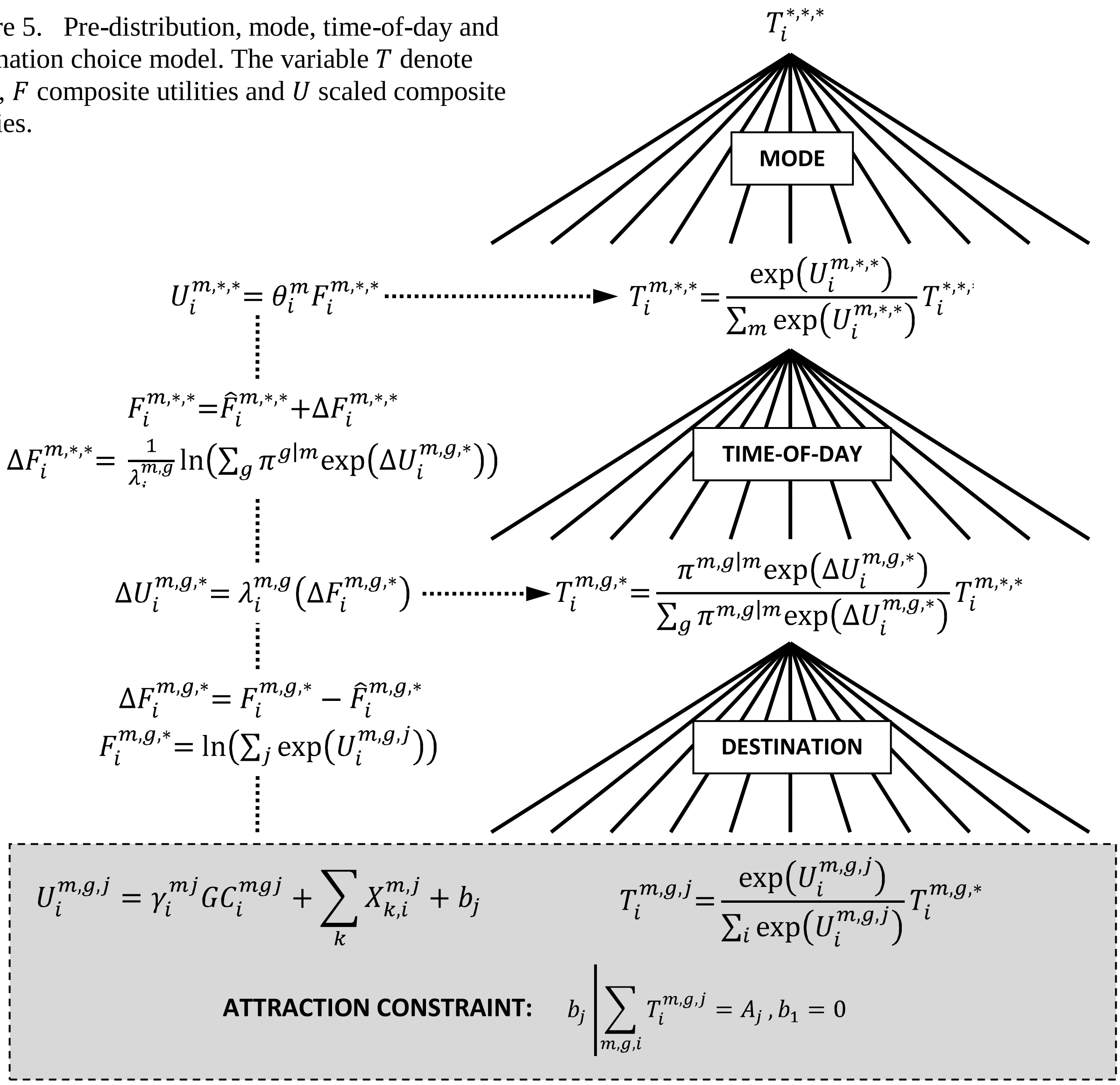


Figure 5. Pre-distribution, mode, time-of-day and destination choice model. The variable $T$ denote tours, $F$ composite utilities and $U$ scaled composite utilities.

In general, DMS model sensitivity parameters are calibrated using maximum likelihood techniques that rely on cross-sectional variations in base data. It is unlikely that sufficient cross-sectional variation with regard to cost by tour group (i.e. cost by ToD) will exist in this data to allow for the calibration of MTD models using the same approach. An appropriate methodology for obtaining these parameters (as used for ART3) is as follows; firstly, a conventional DMS model (i.e. without ToD) is estimated using the base data. The parameters of this model are then used in the MTD model for the mode and destination splits[2]. The process used to calibrate the sensitivity of the model to variations in cost by ToD involves 'manually' tuning the relevant parameter(s). A separate discussion regarding this process is presented in section V.

The functional form of both the mode and distribution components of the MTD model is that of two absolute logit models. As the sensitivity parameter for the tour group split is known with less certainty, it is appropriate that this split be implemented as an incremental rather than an absolute logit model. Additional complication arises in the implementation of this model as a consequence of 'inserting' an incremental logit model between two absolute logit models (see Appendix B1).

## V. MODEL CALIBRATION

As discussed in the previous section, the maximum likelihood technique used to calibrate the sensitivity parameters for the mode and destination splits of the MTD model is unlikely to enable calibration of the ToD split as a consequence of limitations in survey data. Consequently, an alternative approach to this calibration is required.

The ultimate goal of ToD choice calibration is to ensure that the sensitivity of the modelled ToD split with respect to differential changes in cost by ToD reflects the behaviour expected by the modelled population. The first task in calibration is thus to gain a quantitative measure regarding this behaviour, the most natural measure being the sensitivity of the proportion of travel in the peak periods to changes in the differential cost of travel between periods. Having done this, the ToD sensitivity parameters may be adjusted such that model results agree with this measure.

For the calibration of the ToD component of the ART3 model, a review of international findings pertaining to the sensitivity of ToD travel choices was conducted. Material reviewed regarded both peak-contraction as a result of the provision of extra capacity in congested contexts (e.g. Lian, 2005 and Bly, 2005) and of peak spreading following differential pricing initiatives (e.g. Kroes et al., 1996). The result of this review was an estimate of the elasticity of peak period traffic share to change in peak period generalised cost of -0.5. The ART3 sensitivity parameters were adjusted such that this elasticity was reflected by the model.

Having calibrated the ART3 model ToD choice sensitivity to international findings, an attempt was made to validate this sensitivity using data specific to Auckland. An analysis of historical screenline data for Auckland suggested an elasticity of peak period travel flow to change in total traffic flow of the order of 0.2. It is important to note that this figure is in no way comparable to the value of -0.5 discussed above, as it regards a different measure of sensitivity. Furthermore, this elasticity is only indicative as the observed changes in traffic flow over the observed period cannot be attributed to differential changes in cost of travel by ToD alone. Scenario testing of the ART3 model implied an equivalent elasticity of 0.16, comparable with the results of the historical analysis.

Further to the above validation, a road user charging scenario was conducted to ensure the model produced sensible results. A monetary charge was applied to travel across a cordon around the Auckland Isthmus. The charge was applied in the peak direction during the peak periods, that is, inbound in the AM

[2] The MTD model represents travel using a tour basis. An average tour will experience twice the cost of travel of an average trip. Consequently, if the DMS model is estimated on a trips basis, the sensitivity parameters used for the MTD model will be ½ of those estimated to ensure the mode and destination split sensitivities of the models are equivalent.

peak and outbound in the PM peak. As a result of this charge, vehicle volumes crossing the cordon in the charged period/directions were predicted to decrease significantly. A smaller decrease in the non-priced period/directions was also observed as a consequence of the tour-based nature of the choice model.

## VI. Conclusions

In this paper, a methodology for the modelling of ToD choice within strategic transport models has been presented. In principle, this methodology transforms a conventionally estimated distribution and mode choice model system into a mode, time-of-day and destination choice model via the insertion of a ToD split. This methodology has been successfully implemented and is currently being used in the Auckland Regional Council's ART3 model. In principle, the procedure presented in this paper could be added to existing multimodal transport models without the need for re-estimation, although the software code would require significant amendment.

## Acknowledgemnt

The authors would like to thank Sinclair Knight Merz Ltd for encouraging continued development in the field of strategic transport modelling and the Auckland Regional Council (ARC) for allowing the publication of the results presented in section V.

## References

Ortúzar, J., & L. Willumsen. *Modelling Transport – 3rd Edition*. Chinchester, UK: John Wiley & Sons Ltd, 2001.

Booz Allen Hamilton, *Transfund Research Report 241 - Research into Peak Spreading*, 2003.

WebTAG Unit 3.12.2; *http://www.webtag.org.uk/*

*webdocuments/3_Expert/12_Road_Pricing/pdf/3.12.2-consult.pdf*, accessed July 2008

WebTAG Unit 3.10.3; *http://www.webtag.org.uk/*

*webdocuments/3_Expert/10_Variable_Demand_Modelling/3.10.3.htm*, accessed July 2008

Gordon, A., M. MacDonald, A. Daly, J. Bates, *Modelling time period choice in large-scale hierarchical demand models: some problems and a* solution, Proceedings of the Eurpoean Transport Conference, 2007.

Greene, W. H., *Econometric Analysis – 6th Edition*. Pearson Prentice Hall, New Jersey, 2008.

Lian, J. I., *Impact of Main Road Investments in Bergen and Olso*, Proceedings of the 45th congress of European Regional Science Association, 2005.

Bly, P. H., *Response of Traffic to Increased Road Capacity - A Review of Seven Before-and-After Studies,* UK DfT– Research and development guidance, 2005.

Kroes, E. A., Daly, H. Gunn, T. V. D. Hoorn, *The Opening of the Amsterdam Ring Road*, Transportation Vol 23, 1996.

## APPENDIX A1 – TOUR GROUP COST CONSTRUCTION

The conversion of generalised cost from an OD trip basis to a PA tour basis is done separately for each purpose[3]. This conversion is driven by the following set of conditional probabilities, as derived from base year data:

$\vec{\pi}_{i,j}^{t|m,d}$ : For trips originating in zone $i$, destined to zone $j$, the conditional probability that a mode $m$, direction $d \in \{\text{fh,th}\}$ trip will occur in time period $t$.

$\pi^{d|m}$ : The conditional probability that a mode $m$ trip will be made in direction $d \in \{\text{fh,th}\}$.

Before proceeding further we note that the approximation $\pi^{d|m} = 0.5$ is valid only if travel behaviour may be approximated as symmetric within purpose-mode segments, that is, if the travel data suggests a one-to-one mapping between outbound and return trips of the same purpose and mode. Such symmetry is by no means gauranteed.

Define the variable: $\overrightarrow{\mathrm{GC}}_{\mathrm{i,j}}^{\mathrm{m,t}}$, denoting the generalised cost of travel for trips by OD zone and time period (i.e. the assignment quantity). For tours, the generalised cost of travel is calculated by direction $d \in \{\text{fh,th}\}$ as a weighted average of these costs:

$$\overrightarrow{\mathrm{GC}}_{i,j}^{m,g,d} = \frac{\sum\limits_{t \in d \text{ legs of } g} \vec{\pi}_{i,j}^{t|m,d} \overrightarrow{\mathrm{GC}}_{i,j}^{m,t}}{\sum\limits_{t \in d \text{ legs of } g} \vec{\pi}_{i,j}^{t|m,d}} \quad , \qquad d \in \{\text{fh,th}\}. \tag{A1}$$

The total tour cost is obtained as the sum of the costs in each direction, weighted to account for asymmetry:

$$GC_i^{m,g,j} = 2 \sum_d \pi^{d|m} \overrightarrow{\mathrm{GC}}_{i,j}^{m,g,d} , \qquad d \in \{\text{fh,th}\}. \tag{A2}$$

It is clear that the conditional probabilities: $\pi$ and $\vec{\pi}$ will vary from the base to the forecast year as a consequence of changes in ToD travel choices. However, the influence of this change on the ToD choice model (i.e. via the construction of tour group costs) is secondary to the effect of the ToD choice model itself.

## APPENDIX A2 – TOUR GROUP TO TRIP CONVERSION

The mapping of tours by tour-group, mode and PA zone to trips by time period, mode and OD zone is achieved by weighted averaging of tour groups as follows; firstly, tours are disaggregated by direction $d \in \{\text{fh,th}\}$ as follows:

$$\vec{\mathrm{T}}_{i,j}^{m,g,d} = \pi^{d|m} T_i^{m,g,j}, \qquad d \in \{\text{fh,th}\}. \tag{A3}$$

The next step requires the conditional probabilities of a trip occurring in each time period given its tour group. These probabilities are calculated from those probabilities derived from the base data (i.e. $\vec{\pi}_{i,j}^{t|m,d}$ and $\pi^{d|m}$ , see Appendix A1) as follows:

3 However, for readability, our notational convention is to omit subscripts denoting purpose.

$$\vec{\pi}_{i,j}^{t|g,m,d} = \begin{cases} \dfrac{\vec{\pi}_{i,j}^{t|m,d}}{\sum_{t \in d \text{ legs of } g} \vec{\pi}_{i,j}^{t|m,d}} & \text{if } t \in d \text{ legs of } g \\ 0 & \text{if } t \notin d \text{ legs of } g \end{cases}, \qquad d \in \{\text{fh,th}\}. \tag{A4}$$

Using these values, tours by tour group, mode and direction are mapped to trips by time period, mode and direction as follows:

$$\vec{\mathrm{T}}_{i,j}^{m,t,d} = \sum_{g} \vec{\pi}_{i,j}^{t|g,m,d} \, \vec{\mathrm{T}}_{i,j}^{m,g,d}, \qquad d \in \{\text{fh,th}\}. \tag{A5}$$

Summing across directions, we arrive at trips by mode and time period:

$$\vec{\mathrm{T}}_{i,j}^{m,t} = \sum_{d \in \{\text{fh,th}\}} \vec{\mathrm{T}}_{i,j}^{m,t,d} \tag{A6}$$

## APPENDIX B1 - MODE, TIME-OF-DAY AND DESTINATION CHOICE MODEL

Figure 5 illustrates a complete description of the of the MTD model. This appendix is intended to augment this figure.

The order of execution of the various components of the model is as depicted in figure 5. That is, (a) the composite costs are calculated for each split, (b) the splits are performed, (c) the attraction constraint parameters are updated and (d) the process is repeated until convergence. Rather than discussing the details of this execution in a chronological fashion, we address the four principle elements of the model (i.e. mode split, time-of-day split, destination split and attraction constraint) individually within the following sub-sections.

To aid in explanations to follow, at this point it is useful to make the following variable definition:

$T_i^{m,g,j}$ : Tours by production zone i, mode m, tour group g and destination zone j.

The superscript $*$ is used to denote summation. Equivalent notation is used for utilities, scaled utilities and scaled composite utilities.

### A. *Mode Split*

The mode split component of the MTD model splits tours by production zone: $T_i^{*,*,*}$ into tours by production zone and mode: $T_i^{m,*,*}$. It achieves this via the application of the logit model:

$$T_i^{m,*,*} = \frac{\exp\left(U_i^{m,*,*}\right)}{\sum_m \exp\left(U_i^{m,*,*}\right)} T_i^{*,*,*}, \tag{A7}$$

were $U_i^{m,*,*}$ is the scaled composite utility:

$$U_i^{m,*,*} = \theta_i^m F_i^{m,*,*}. \tag{A8}$$

Here, the scaling parameter $\theta_i^m$ represents the relative sensitivity between the mode and destination splits (as derived from the calibration of a DMS model). The composite utilities: $F_i^{m,*,*}$ are derived via the time-of-day split.

### B. *Time-of-day Split*

The time-of-day split component of the MTD model splits tours by production zone and mode: $T_i^{m,*,*}$ into tours by production zone, mode and tour group: $T_i^{m,g,*}$. It achieves this via the incremental logit model:

$$T_i^{m,g,*} = \frac{\pi^{g|m} \exp\left(\Delta U_i^{m,g,*}\right)}{\sum_g \pi^{g|m} \exp\left(\Delta U_i^{m,g,*}\right)} T_i^{m,*,*}, \tag{A9}$$

where $\pi^{g|m}$ denotes the base probability that a tour of mode $m$ will be made in tour-group $g$ (i.e. the base proportion of trips by tour group for each mode), and $\Delta U_i^{m,g,*}$ is the scaled composite utility difference:

$$\Delta U_i^{m,g,*} = \lambda_i^{m,g}\left(\Delta F_i^{m,g,*}\right). \tag{A10}$$

The scaling parameter $\lambda_i^{m,g}$ represents the relative sensitivity between the destination and tour group splits. In principle, this parameter could be made to vary by mode, tour-group and production zone. However the manner in which it is calibrated renders this unnecessary, and in general: $\lambda_i^{m,g} = \lambda \; \forall, g, i$. The composite utility difference $\Delta F_i^{m,g,*}$ is calculated as:

$$\Delta F_i^{m,g,*} = F_i^{m,g,*} - \hat{F}_i^{m,g,*}, \tag{A11}$$

where $\hat{F}_i^{m,g,*}$ are the base composite utilities and $F_i^{m,g,*}$ the scenario composite utilities, derived via the destination split component of the model.

For the purposes of the mode-split component of the model, the following composite utilities are computed:

$$\Delta F_i^{m,*,*} = \frac{1}{\lambda_i^{m,g}} \ln\left(\sum_g \pi^{g|m} \exp\left(\Delta U_i^{m,g,*}\right)\right), \tag{A12}$$

These are incremental composite utilities and as such must be added to the base composite utilities: $\hat{F}_i^{m,*,*}$ prior to being used in the mode split model; this model having the functional form of an absolute logit model. That is:

$$F_i^{m,*,*} = \hat{F}_i^{m,*,*} + \Delta F_i^{m,*,*}. \tag{A13}$$

The observant reader will have noticed the additional scaling used in the construction of the composite utilities defined by equation A12. This scaling is necessary to preserve the relative sensitivity between the mode and destination splits (i.e. to ensure this relative sensitivity is not 'upset' by the introduction of the tour group split).

C. *Destination Split*

The destination split component of the MTD model splits tours by production zone, mode and tour group: $T_i^{m,g,*}$ into tours by production zone, mode, tour group and destination zone: $T_i^{m,g,j}$ as follows:

$$T_i^{m,g,j} = \frac{\exp\left(U_i^{m,g,j}\right)}{\sum_i \exp\left(U_i^{m,g,j}\right)} T_i^{m,g,*}. \tag{A14}$$

were $U_i^{m,g,j}$ is the utility of travel by mode, tour group and PA movement, computed as:

$$U_i^{m,g,j} = \gamma_i^{mj} GC_i^{m,g,j} + \sum_k X_{k,i}^{m,j} + b_j. \tag{A15}$$

Here, $\gamma_i^{mj}$ are a set of parameters defining the sensitivity of the model with respect to generalised cost of travel. The variables $X_{k,i}^{m,j}$ are set of $k$ constants representing travel attributes not encapsulated in the generalised cost (e.g. comfort and safety). Both these parameters are derived from the calibration of a conventional DMS model and thus may vary by mode and trip geography (i.e. PA zone), but not tour group. As introduced in the main text, $b_j$ are a set of destination specific constants used to ensure the attraction constraint is satisfied. The exact value of these parameters is arrived at via iteration, as discussed in the next sub-section.

For the purposes of the time-of-day component of the model, the following composite utilities are computed:

$$F_i^{m,g,*} = \ln\left(\sum_j \exp\left(U_i^{m,g,j}\right)\right). \tag{A16}$$

### D. *Attraction Constraint*

Upon the first iteration of the MTD model, the tours by mode, time-of-day and destination resulting from the splits discussed above will in general not satisfy the attraction constraint:

$$\sum_{m,g,i} T_i^{m,g,j} = A_j. \tag{A17}$$

To address this, the destination specific parameters $b_j$ are adjusted iteratively such that this constraint is asymptotically satisfied. Defining $[k]$ as the iteration number, at each iteration these parameters are updated via the feedback law:

$$b_j^{[k+1]} = \begin{cases} b_j^{[k]} \dfrac{A_j}{\tilde{A}_j^{[k]}} & \text{if } j \neq 1 \\ 0 & \text{if } j = 1 \end{cases} \tag{A18}$$

where $\tilde{A}_j^{[k]}$ are the synthesizesd attractions arrising from iteration $[k]$. That is:

$$\tilde{A}_j^{[k]} = \sum_{m,g,i} T_i^{m,g,j} \bigg|_{b_j^{[k]}}. \tag{A19}$$

It is a straightforward exercise to demonstrate that the dynamics resulting from this iterative process are exponentially stable.

As specified in equation A18: $b_1^{[k]} = 0 \; \forall [k]$. The rationale for this is as follows; the destination split is driven by the absolute differences between the utilities of travel to the alternative destinations. As such, if all destination specific constants were allowed to vary, they would have one additional degree of freedom. This is not a problem in itself, however it will introduce problems if convergence tests utilise these parameters. The decision to fix $b_1^{[k]} = 0 \; \forall [k]$ removes this degree of freedom. In principle, this condition specifies the cost of travel to zone 1 as a reference, although the decision to use this specific zone is an arbitrary one.