

User-operator Based Model for Optimal Scheduling of Public Transport Systems

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This paper deals with the development of a strategic approach for optimising the operation of a public transport system that considers both the user's objective and the operator's objective. Passengers of public transport are assumed to seek minimum wait times to conduct trips, while on the other hand, operators are concerned with the efficient operation such as minimum fleet size. The average minimum wait time is to be achieved by creating an optimal despatching policy of each vehicle from the terminal. As for efficient operation the utilisation of vehicles should be maximised by having a minimum number of vehicles in operation. User's and operator's objectives are optimised within certain operational constraints such as vehicle capacity to maintain an acceptable level of service. The model is constructed in a bi-level programming form in which the user's objective is minimised by dynamic programming and the operator's objective is minimised by dead-heading strategy. Furthermore, an algorithm and a contrived example are developed to solve and observe the performance of the approach.

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OF PUBLIC TRANSPORT SYSTEMS**

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Abstract : This paper deals with the development of a strategic approach for optimizing the operation of a public transport system that considers both the user's objective and the operator's objective. Passengers of public transport are assumed to seek minimum wait times to conduct trips, while on the other way, operators are concerned with the efficient operation such as minimum fleet size. The average minimum wait time is to be achieved by creating an optimal dispatching policy of each vehicle from the terminal. As for efficient operation the utilisation of vehicles should be maximised by having a minimum number of vehicles in operation. User's and operator's objectives are optimized within certain operational constraints such as vehicle capacity to maintain an acceptable level of service. The model is constructed in a bi-level programming form in which the user's objective is minimized by dynamic programming and the operator's objective is minimized by dead-heading strategy. Furthermore, an algorithm and a contrived example are developed to solve and observe the performance of the approach.

Introduction

Public transport is defined as a public service and it should be, in general, providing service that complies with public demand. However, in practice to create a public transport system that would comply with the level of service which is still within the tolerance of users may appear costly. The system may even cost more when it is related to the policy of shifting travel demand from the private transport system to the public transport system. There have been a lot of public transit companies that have to sacrifice their performance or level of service due to their limited resources, or they will go bankrupt. So it should be of primary concern that the public transport system is to be established under the perception of satisfying objectives of both users and operators.

There has been much research aimed at partially solve the problem. Some of them try to solve in the operator's perspectives such as optimal routing that covers the maximum number of user's, such as Salzborn (1972) and Hurdle (1973). In a different manners some try to solve in user perspectives such as optimal dispatching policy that minimizes total wait time, see Newell (1971), Chapman & Mitchel (1978) and Sutanto (1989). Jordan & Turnquist (1979) for example, investigated the relationship between delay at stop and number of boarding passengers which is an influencing issue for scheduling. Consideration of load factors for

vehicles on each link was also analyzed by Ceder (1984), Ceder & Wilson (1986) and LeBlanc (1988). Optimal routing strategy that minimizes the total number of transferring passengers as well as total travel cost at network level was intensively investigated by Sutanto (1992). However, a case can be regarding a solution to the problem that considers both user's and operator's objectives. This is the aim of this research since it is very obvious that no public transport system will be sustainable and economically viable if it is developed based on a partial approach, such as user perspectives or operator's perspectives only.

The ensuing sections are arranged to explain the whole concept from model development to conclusions drawn. Section 2 identifies problems and they are represented in the mathematical programming form. Section 3 explains how the model developed in section 2 is solved and concluded with a proposed algorithm. To see the performance of the proposed algorithm section 4 provides illustration with a contrived small problem. Finally section 5 concludes the findings and directions for future research.

Model development

As already explained above, this research approaches the problem by determining both user's objectives and operator's objectives. The concept of optimizing both objectives can be comprehended by total system as illustrated by Figure 1. The system may have several routes with different terminals and coincident terminals.

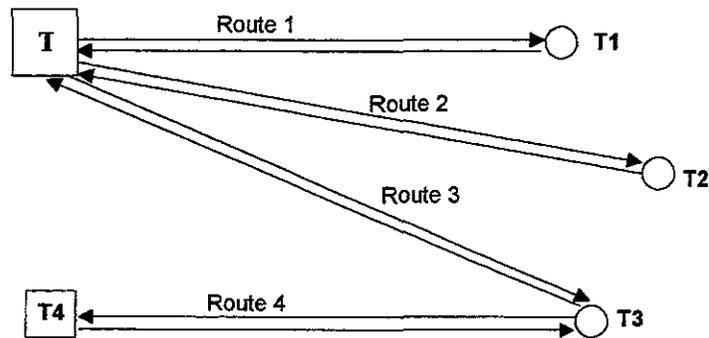


Figure 1. Public Transport Routes

Minimum wait time

Objective function :

If one route is detached and augmented, it consists of several stops that allow passengers to board and alight such as illustrated in Figure 2. By knowing some information on passenger arrival at each stop which is time dependent, and travel time of the transit vehicle from one stop to the other, total waiting time of all passengers along the public transit route can be

minimized. This minimization problem may further be constrained by vehicle capacity for the sake of a specific tolerable service level

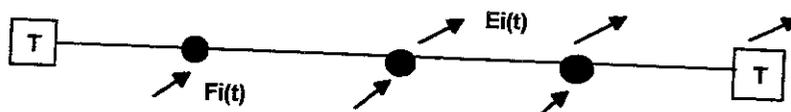


Figure 2. Boarding and Alighting Passengers

If $F_i(t)$ denotes cumulative number of passengers which is time dependent at stop and t_{ij} denotes arrival time of i th vehicle at stop j , the total wait time of all passengers at stop i , W_i , can be given as;

$$W_i = \sum_{j=1}^N \int_{t_{j-1}}^{t_{ij}} [F_i(t) - F_i(t_{j-1})] dt \quad \forall i \dots (1)$$

where N is total number of vehicles on the route

The dispatching policy can actually be made easier if all arrival functions along the route can be shifted to the departure point, namely terminal. This can be done if all travel times from one stop to the next are known. If Δt_k shows travel time from stop k to the next, the shifted total arrival functions can be given as;

$$F(t) = \sum_{i=1} F_i(t + \sum_k \Delta t_k) \dots (2)$$

By having the formulae given in equations (1) and (2) the optimal policy of vehicle departure can be made as illustrated in Figure 3. The policy is supposed to create vehicle departure that minimizes total wait time of all passengers, or in other words to minimize the area between arrival function and departure step function shown in Figure 3

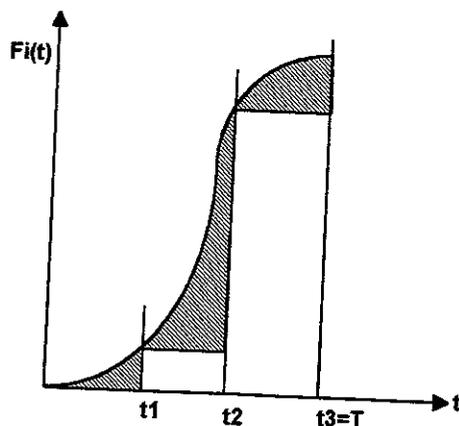


Figure 3. Optimal Strategy of Vehicle Departure

Constraint :

Once the objective function of minimizing total wait time is determined the next step is to determine the above-mentioned constraint that relates to the achievement of the specified service level. One of the most common indicators of passenger's preference to use public transit is a guarantee of having a seat. It is then necessary to limit number of passengers to the vehicle capacity, C . To know the number of passengers a board of vehicle is easy when the number of alighting and boarding passengers at each stop is known. These numbers can be determined when O-D transition probability of travelling passengers at each stop is given. If R_{il} denotes the O-D transition probability of travelling passengers from stop i to stop l , the total number of passengers on board of vehicle j at stop i can be formulated as;

$$D_i(t_j) = D_{i-1}(t_j + \Delta t_0) + F_i(t_j + \Delta t_0) - E_i(t_j) \quad (4)$$

Where $E_i(t_j)$ denotes number of alighting passengers at stop i from vehicle j is given as;

$$E_i(t_j) = \sum_{l=1}^{i-1} R_{il} [F_l(t_{ij}) - F_l(t_{i,j-1})] \quad (5)$$

$$\sum_l R_{il} = 1 \quad \forall i \quad (6)$$

and t_{ij} is arrival time of vehicle j at stop i . It is clear that the constraint sought is similar as to redefine equation (4) as follows;

$$D_i(t_j) \leq C \quad \forall i, j \quad (7)$$

Considering the objective function and its constraint the optimal scheduling of vehicles to serve the passengers within a certain frequency or number of vehicles can be rewritten as (Sutanto, 1989);

$$\text{Min } Z_{1ij} = \sum_j \sum_i \int_{t_{i,j-1}}^{t_{ij}} [F_i(t) - F_i(t_{ij})] dt \quad (8a)$$

Subject to,

$$D_i(t_j) \leq C \quad \forall i, j \quad (8b)$$

$$t_1 < t_2 < \dots < t_N = T \quad (8c)$$

where, N is total frequency or number of despatching vehicles on the route and T is time period of schedule block under consideration. It is shown further by Sutanto (1989) that such optimization problem can be solved efficiently by *Dynamic Programming* (DP). Headway of vehicle, namely s'_j , can be chosen as decision variables, while the stage corresponds to each dispatched vehicle and state variables, namely Q'_j 's, are the time interval between the beginning of time period of interest and the j th despatch. The recursive relationship of this problem in the dynamic programming solution is given by,

$$r_j(Q_j) = \text{Min}_{s_j} [r_{j-1}(Q_{j-1}) + f_j(s_j)] \dots (9)$$

where $r_j(Q_j)$ is the minimum wait time for j vehicles from the first, and $f_j(s_j)$ is the wait time for the j th vehicle. The state of dynamics is given by,

$$Q_j = Q_{j-1} + s_j \dots (10)$$

Optimal fleet size

By this end the problem of setting optimal scheduling is obtained. It is, however, an optimality achieved when only the user's objective is considered in which the number of vehicles (fleet size) may not necessarily be optimal. It is then the next search to be found that when all time departures of vehicles are set for each route, is it possible to obtain the least number of vehicles in use by arranging the vehicle movements from one route to the other and so forth. In other words the model is going to be developed further by accommodating the possibility of interlining.

Problem definition

If timetables of arrivals and departures of all routes are obtained, they may comprise of certain required number of vehicles. The problem to be solved further is how to develop chains of passages from those timetables that minimizes the fleet size.

Definitions :

The problem stated above is quite straight-forward and can be represented in a optimal network problem. A bipartite network can be developed which contains two sets of nodes. One node set is supposed to comprise of arrival times, and the other node set comprises of departure times. By this representation it is possible to use *Maximum Cardinality Matching* (MCM) method to solve. However, prior to proceeding to the solution several definitions need to be clarified such as followings;

1. Passage is a 4 tuple $\mathbf{p} = (p_1, p_2, p_3, p_4)$
 - p_1, p_2 represent terminals of departure and arrival respectively
 - p_3, p_4 are real numbers such that $0 \leq p_3 \leq p_4$
 - p_3 is the departure time (from p_1)
 - p_4 is the arrival time (to p_2)
2. A Chain is a finite or infinite sequence of passages which might be performed by one vehicle
3. A Fleet is a partition of the schedule into chains

4. Bipartite Network is a network whose node set can be partitioned into two subsets x' & x'' such that no link in the network joins two nodes in the same subset
5. A Matching is any set of links in a network such that each node of the network is incident to at most one link in this set.
6. Maximum Cardinality Matching is a matching that contained the greatest possible of links

Problem formulation :

The problem of finding the efficient chains of passages can be modeled as the problem of finding MCM from a bipartite network whose subsets represent arrival and departure time

The bipartite network (BN) consists of two subsets x' and x'' , where x' and x'' denote arrival and departure time respectively

A couple of nodes represents two passages that could form a chain

Two nodes A_i and $D_j (A_i \in x', D_j \in x'')$ can be joined if the arrival time is earlier than departure time such as;

$$BN(x', x'') = \begin{aligned} & \{(A_i, D_j) \mid D_j \geq A_i \forall i, j \text{ if arr terminal for } A_i = \text{depart terminal for } D_j \\ & \{(A_i, D_j) \mid D_j \geq A_i + t_{ij} \forall i, j \text{ if arr terminal for } A_i \text{ depart terminal for } D_j \end{aligned} \quad \dots (11)$$

where,

- $A_i = i$ - th arrival time in x'
- $D_j = j$ - th departure time in x''
- t_{ij} = deadheading time from A_i to D_j

The former condition ($D_j \geq A_i$) is used for chain which permits no - deadheading trips, while the latter case ($D_j \geq A_i + t_{ij}$) is for dead-heading trips

Solution :

Since a couple of nodes in the bipartite network could form a chain and the objective of the original problem is to minimize fleet size, it is necessary to maximize number of couples of nodes. In order words, the problem of minimizing fleet size can be solved by MCM

In this research the MCM method that solves the particular problem of minimum fleet size is developed within the nature of maximum flow algorithm. Furthermore, the algorithm is developed in the following steps;

Step 1. Direct all possible links from subset x' to x'' to form bipartite network (BN)

Step 2. Number the nodes in subset x' in sequence, starting from the nodes having the least

number of links (regardless the nodes with no link)

Step 3 Number the nodes in subset x'' , starting from the head node of the tail one

Step 4. Construct BN' as follows;

- a) Using BN create subset x' and x'' consists of all the tail nodes and head nodes (regardless linkless nodes), respectively. Create the possible links
- b) Using such BN create a source node s and connect link (s, A_i) from the source to each node $\forall A_i \in x'$
- c) Create a sink node t and connect link (D_j, t) from each $D_j \in x''$ to the sink
- d) Let each link capacity equal 1 and initial flow $f(A_i, D_j) = 0, f(S, A_i) = 0, f(D_j, t) = 0, \forall i,$

Step 5. If $f(A_i, D_j)$ or $f(S, A_i)$ or $f(D_j, t) < c(A_i, D_j)$, let $(A_i, D_j), (S, A_i), (D_j, t) \in I$
 If $f(A_i, D_j)$ or $f(S, A_i)$ or $f(D_j, t) > 0$, let $R(A_i, D_j) = f(A_i, D_j)$ and $(A_i, D_j) \in R$

Step 6 Perform augmented flow algorithm on set I and R as follows;

- a) Label node s
- b) Label the links and nodes in sequence according to the following rules until node has been labeled or no further labeling is possible :
 - If node x is labeled and node y is not labeled, then node y and link (x, y) can be labeled in any of the following;
 - * if $(x, y) \in I$ then node y and link (x, y) can be labeled
 - * if $(y, x) \in R$ then node y and link (x, y) cannot be labeled
 - If node t has been labeled, then there exists a unique chain of labeled link from s to t

This chain is a flow augmenting chain and return to Step 5. Otherwise, if t remains unlabeled after the algorithm terminates, then no flow augmenting chain exists from s to t ; Stop. The current flow will show that each link carries either one flow unit or no flow units.

The links from x' to x'' in BN' that carry one flow unit correspond to a matching in BN . Figure 4 illustrates further how the algorithm developed above solves the bipartite network

The explanation above completes the model of minimizing fleet size or maximizing vehicle utility which is the concern of operator. Minimizing the fleet size is made possible by developing proper coordination between arrivals and departures of vehicles among routes such as interlining and dead-heading.

User-operator based model

Having two objectives from different perspectives, the next step is to re-define both into one objective function with certain weights and constraints. Since both objectives are to be minimized, the so called user-operator based model is easily re-defined as following mathematical programming;

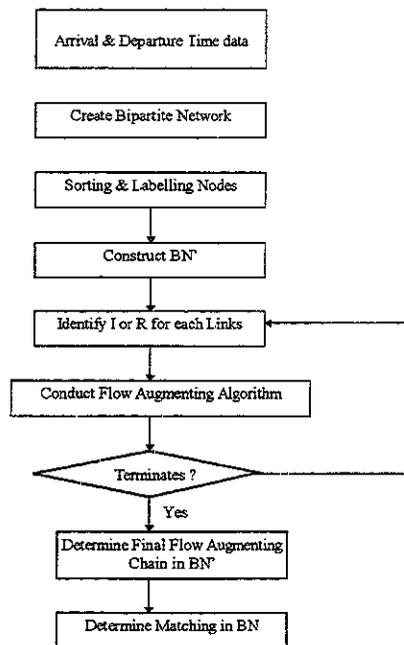


Figure 4. Modified Maximum Cardinality Algorithm

$$\text{Min } Z(Z1, Z2) = \alpha Z1 + \beta Z2 \quad (12)$$

or

$$\text{Min } \sum_{i,j,r} Z(W, FS) = \alpha [\sum_i \sum_j \int_{t_{i-1}}^{t_i} F_i(t) - F_i(t_{ij})] dt + \beta FS[DH(r)] \quad (13a)$$

Subject to

$$D_i(t_j) \leq C \quad \forall i, j \quad (13b)$$

$$t_1 < t_2 < \dots < t_N = I \quad (13c)$$

$$TD_{lmn} \geq TA_{jki} \quad \forall k = l \quad (13d)$$

$$DH(r) = TD_{lmn} \geq TA_{jki} + t_{DH,kl} \quad \forall k \neq l \quad (13e)$$

Where $FS[DH(r)]$ denotes the number of fleet size that can be minimized by deadhead conditions, $DH(r)$, as function of route r given in equation (13d) and (13e). α and β denote the weights of each component of objective function.

The proposed algorithm

The mathematical programming given in equations (13)'s can be solved efficiently by iterative bi-level procedure that combines the solution of optimal scheduling by dynamic programming

and the solution of optimal fleet size determination via deadhead by modified maximum cardinality method developed in the prior sections. Furthermore, Figure 5 illustrates the iterative flow of problem solving for the problem

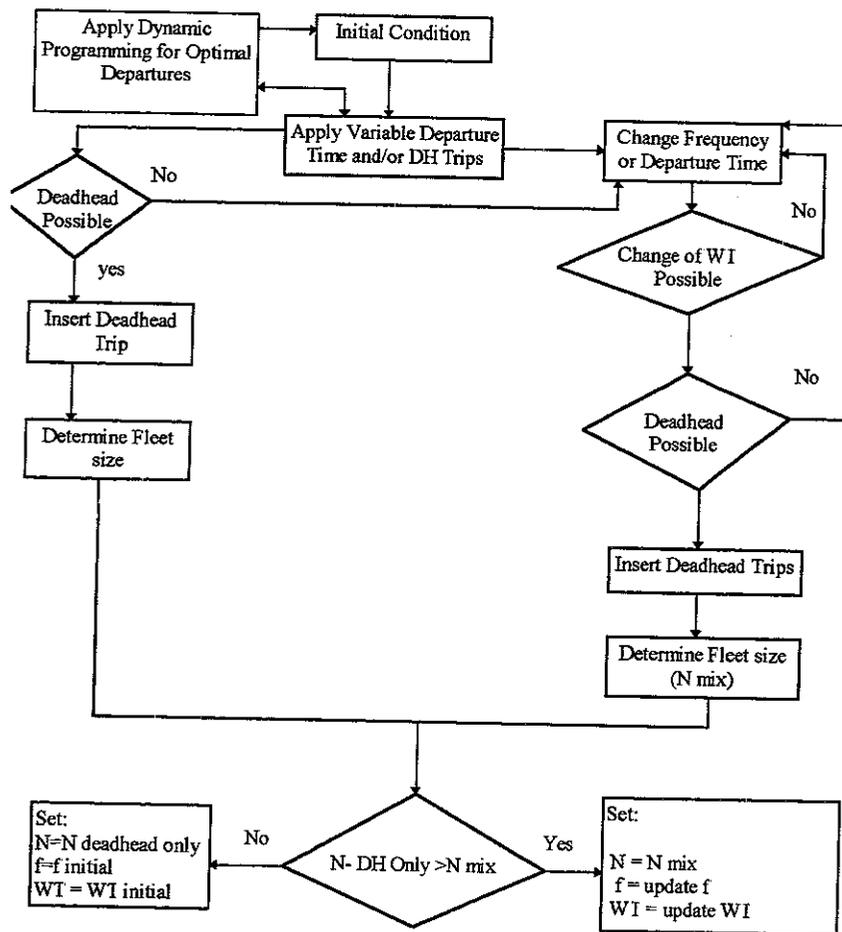


Figure 5. Flowchart of Problem Solving

The algorithm developed above works with objective function determined in equation (13). Optimal scheduling of vehicles for each route is solved by using the dynamic programming. Results of such scheduling produce a set of vehicle arrivals and vehicle departures in which at the first iteration stated as initial condition. Starting with this initial condition an optimization process which minimizes fleet size follows. The process may have to minimize the number of vehicles by changing the value of wait time either in decreasing or increasing manner, and whether it makes the objective function worse-off or better-off, further settings to schedule are up-dated. The process continues until it converges to no better value of objective function.

Example

Problem

To see how the proposed model can be applied contrived example is developed as follows. A depot T is supposed to have 3 rounding routes namely TA/AT (route 1), TB/BI (route 2) and TC/CI (route 3) as illustrated by Figure 6

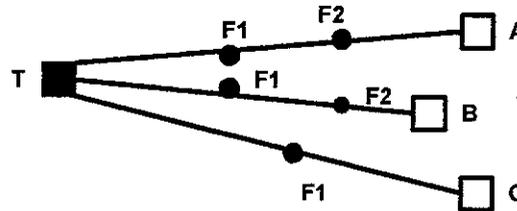


Figure 6. Example of Three Public Transit Routes

Functions of passenger arrival at each stop of the route are given as follows;

Route 1 (TA Direction)

$$F_1(t) = \begin{cases} 0 & 0 \leq t \leq 2 \\ \frac{10}{60}(t-2) & 2 \leq t \leq 62 \\ 15 & t \geq 62 \end{cases}$$

Route 1 (AT Direction)

$$F_1(t) = \begin{cases} 0 & 0 \leq t \leq 5 \\ \frac{15}{60^2}(t-5)^2 & 2 \leq t \leq 65 \\ 15 & t \geq 65 \end{cases}$$

$$F_2(t) = \begin{cases} 0 & 0 \leq t \leq 5 \\ \frac{20}{60}(t-5) & 5 \leq t \leq 65 \\ 20 & t \geq 65 \end{cases}$$

$$F_2(t) = \begin{cases} 0 & 0 \leq t \leq 2 \\ \frac{10}{60^2}(t-2)^2 & 2 \leq t \leq 62 \\ 10 & t \geq 62 \end{cases}$$

with O-D Transition Probability

O/D	2	A
1	0.6	0.4
2	-	1.0

O/D	1	T
2	0.5	0.5
1	-	1.0

and link travel time $\Delta t_0 = 2'$, $\Delta t_1 = 3'$, $\Delta t_2 = 2'$

Route 2 (TB Direction)

$$F_1(t) = \begin{cases} 0 & 0 \leq t \leq 4 \\ \frac{15}{60}(t-4) & 4 \leq t \leq 64 \\ 15 & t \geq 64 \end{cases}$$

$$F_2(t) = \begin{cases} 0 & 0 \leq t \leq 6 \\ \frac{20}{60}(t-6) & 6 \leq t \leq 66 \\ 20 & t \geq 66 \end{cases}$$

Route 2 (BT Direction)

$$F_1(t) = \begin{cases} 0 & 0 \leq t \leq 2 \\ \frac{12}{60^2}(t-2)^2 & 2 \leq t \leq 62 \\ 12 & t \geq 62 \end{cases}$$

$$F_2(t) = \begin{cases} 0 & 0 \leq t \leq 4 \\ \frac{12}{60^2}(t-4)^2 & 4 \leq t \leq 64 \\ 12 & t \geq 64 \end{cases}$$

with O-D Transition Probability;

O/D	2	B
1	0.4	0.6
2	-	1.0

O/D	1	I
2	0.75	0.25
1	-	1.0

and link travel time $\Delta t_0 = 4'$, $\Delta t_1 = 2'$

Route 3 (TC Direction)

$$F_1(t) = \begin{cases} 0 & 0 \leq t \leq 2 \\ \frac{20}{60}(t-2) & 2 \leq t \leq 62 \\ 20 & t \geq 62 \end{cases}$$

Route 3 (CT Direction)

$$F_1(t) = \begin{cases} 0 & 0 \leq t \leq 6 \\ \frac{10}{60^2}(t-4)^2 & 6 \leq t \leq 66 \\ 10 & t \geq 66 \end{cases}$$

with O-D Transition Probability;

O/D	1	C
1	-	1.0

O/D	1	I
1	-	1.0

and link travel time $\Delta t_0 = 2'$, $\Delta t_1 = 4'$

While deadhead trip times are given as follows;

O/D	T	A	B	C
T	-	2	2	3
A	2	-	1	2
B	2	1	-	2
C	3	2	2	-

Table 1. Deadhead Trip Times

and weight values such as $\alpha = 0.01$, $\beta = 2.0$

Assignment

Find the best scheduling that minimizes the objective function determined in equation (13) in which total waiting time and number of fleet size are the control variables. Deadheading and interlining (deadhead with zero deadhead time) are allowed.

Solution

Step 1.

The problem is firstly solved through finding the optimal departure time of each vehicle from the terminal. This is done by utilizing the dynamic programming approach as explained above in the proposed algorithm, in which the results are further used as initial conditions. A summary of such results is given in the following table;

ROUTE	t1	t2	t3	WAIT TIME (minutes)
I TA	20'	40'	60'	450'
AT	42.5'	60'	-	274.13'
II TB	20'	40'	60'	383.4'
BT	40'	60'	-	266.67'
III TC	30'	60'	-	300'
CT	60'	-	-	200'
Total Wait Time				1874.2'

Table 2. Time Schedules & Wait Times

This schedule scenario results in 6 vehicles (fleet size) in operation, total wait time of 1874.2' and objective function value of 30.74 (monetary unit). Figure 7 shows the timetable produced by the initial condition

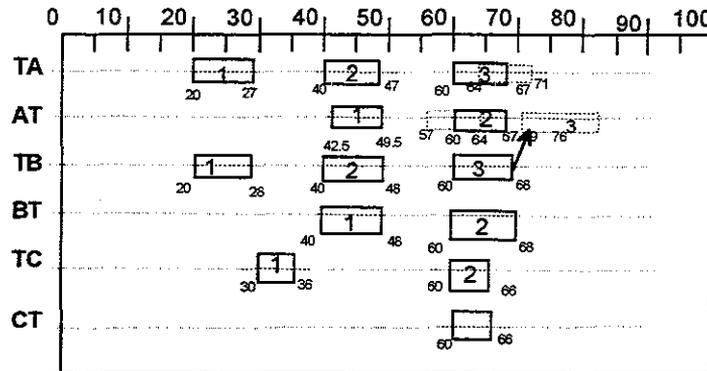


Figure 7. Initial Timetable

Step 2.

Setting time-table and chain of passages. This step is done to seek the possibility of having interlining and/or deadheading vehicles without shifting departure time. This problem is solved by using proposed algorithm in maximum matching of deadheads. In this example a set of matching between subset of departure times and subset of arrival times is made with the following results;

Matching	Chain of Passages
1. TA1 - AT1	1. TA1 - AT1 - TC2
2. IA2 - AT2	2. IA2 - AT2
3. IB1 - BT1	3. IB1 - BT1 - IB3
4. IB2 - BT2	4. IB2 - BT2
5. TC1 - CT1	5. TC1 - CT1
6. AT1 - TC2	6. TC1 - CT1
7. BT1 - IB3	
8. TA3 (single passage)	

Table 3. Sets of Matching & Passage Chains

This effort of matching still does not improve the objective function since no departure time is changed so it produces same total wait time with 6 vehicles (fleet size), and objective function value of 30.74 (monetary unit)

Step 3.

Further improvement is sought by inserting possibility of interlining and/or deadheading that makes any shifting of departure times. This sort of departure shift is introduced when it can improve total wait time. This step is conducted by shifting departure and consequently repeating the procedure done in step 2 for new matching and chains of passages. Shifting departures are done at trip I to A that the 3rd vehicle is dispatched later at 64', and trip A to I the 2nd vehicle is dispatched earlier at 57' in which it requires additional vehicle (the 3rd one) at 69'. This 3rd vehicle is a deadhead from terminal B of arriving vehicle at 68'. The results are as follows;

Matching	Chain of Passages
1. TA2 - AT2	1. TA2 - AT2 - TA3
2. IB2 - BT2	2. IB2 - BT2
3. IC1 - CT1	3. TC1 - CT1
4. AT2 - IA3	4. IA1 - AT1 - TC2
5. IA1 - AT1	5. IB1 - BT1 - IB3 (+ AT3A)
6. IB1 - BT1	
7. AI1 - TC2	
8. BI1 - IB3	

Table 4. Improved Sets of Matching & Passage Chains

This scenario, for the given small example, slightly increases the total wait time from 1874.2' to 1932.4', but substantially decreases the fleet size from 6 vehicles to 5 vehicles. These changes have consequently improved the value of objective function from 30.74 to 29.32

(monetary unit). The relative value of improvement achieved in this example may appear insignificant. However, the absolute value is quite meaningful, and such a situation may even be of greater significance when the real-world problem of a transit system with a large number of vehicles is considered. Furthermore, the results can be summarized in the following table;

Policy	Total Wait Time (minutes)	Fleet Size (vehicles)	Objective Function (monetary unit)
1. Initial Condition	1,874.2	6	30.74
2. Interlining	1,874.2	6	30.74
3. Interlining + Shift Departure Time + Deadheading	1,932.4	5	29.32

Table 5. Summary of Various Policies in The Proposed Model

Conclusions

An optimization model for public transit operation is developed in this research. The proposed model represents the problem on the basis of both user's and operator's objectives. Users are considered to minimize their total wait time, while operators are in general trying to minimize their cost of providing the total number of vehicles or fleet size. Comfort of passengers in the vehicle is also considered and exposed as constraint of which the total number of passengers in each vehicle should be less or equal to vehicle capacity.

The model is formulated in a mathematical programming with objective function that minimizes both total wait time and total number of vehicles in operation. This objective function is further constrained by certain load factor for comfort and other conservation of time schedule. An algorithm is also developed to solve the problem which is based on a bi-level optimization approach. The optimal schedule that minimizes total wait time is solved by using dynamic programming, while the minimum number of vehicles in operation is determined by using modified maximum cardinality method. The two methods interact within the algorithm developed and converge to a certain optimal value of objective function. Computation experience upon small contrived problem shows that the proposed model could solve the problem efficiently.

The capability to representing more realistic assumptions is introduced by the proposed model. Several design parameters in the model are easy to test for elasticities between supply and demand. Furthermore, it is expected that the model is to be developed in a large scale package and possibly to include stochastic characteristics. This development may improve the model substantially and increase the acceptance within the public transit properties.

References

- Ceder, A (1984) Bus Frequency Determination Using Passenger Count Data, *Transportation Research* 18A, No. 5/6.

Ceder, A and Wilson, N H M (1986) Bus Network Design *Transportation Research* 20B, No 4.

Chapman, R A and Michel, J F (1978) Modelling The Tendency of Buses to Form Pairs *Transportation Science*, Vol. 12, No 2.

Hurdle, V F (1973) Minimum Cost Schedules for a Public Transportation Route *Transportation Science*, Vol. 7

LeBlanc, L J (1988) Transit System Network Design *Transportation Research* 22B, No 5

Newell, G F (1971) Dispatching Policies for a Transportation Route *Transportation Science*, Vol 5, No. 1.

Salzborn, F J M (1972) Optimum Bus Scheduling *Transportation Science*, Vol 6, No. 2.

Sutanto, S (1991) Supply Model for Common Bus Routes Under Deterministic Conditions *Proceedings of JSCE - Japan*, No 431, IV-15.

Sutanto, S (1992) Reconnection of Routes in Designing Optimal Bus Network *Proceedings of The 47th JSCE Annual Conference - Japan*