PRIORITY BERTHING IN PORTS AND MULTI-ATTRIBUTE DECISION-MAKING METHODS

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ABSTRACT: *The traditional method of servicing ships that arrive at a port is "first-come, first-served". However, in congested ports, and in situations where certain cargoes are urgently needed for national development projects, the question arises whether a scheme of priority berthing could be more efficient than berthing based on order of arrivals. One of the major problems in the establishment of analytical methods for ports is that few ports, even in the same country, operate under the same conditions. Consequently, the purpose behind developing a computer-based, decision-making model for berthing operations (in a single berth, general use port) was to select a limited number of relatively common objectives in order to optimise the generalised social cost of congestion.*

A multi-attribute decision making framework is outlined. This paper discusses the setting of objectives and the application of weights to system attributes so as to guide the decision maker in selecting the best sequence of servicing vessels. The application of two, slightly different, multiple attribute decision making techniques -- namely SAW and TOPSIS -- are demonstrated with a simple, numerical example, and the order of servicing vessels and costs of delay are compared with the "first come, first served" solution. The algorithms are outlined. The model has been written as a computer program (CONPORT) for use on an IBM personal computer, and its main features are described.

INTRODUCTION

Research into the application of mathematical programming and optimisation methods to the land use/transport system in the Department of Transport Engineering (formerly School of Traffic Engineering), University of New South Wales, now spans three decades. Originally, linear programming techniques were exploited to model the origin-destination desire line pattern of traffic using single objective functions (Blunden, 1971, Chapter 4) but later work incorporated multi-objective programming (Black and Kuranami, 1980; Blunden and Black, 1984, Chapter 4). An extension of this latter work has been in the application of multiple-criteria decision making methods to port operations. Research in the School has also exploited the application of queueing theory and simulation in port planning (Jones and Blunden , 1968; and Buckley and Gooneratne, 1974). However, because there is no forecasting component in our research, except for the estimation of total service time of each individual vessel awaiting berthing, simulation techniques and queueing theory are not used.

Port congestion is an economic cost to the country because any additional shipping turnaround costs (as well as the costs of delay to cargo on board) are normally passed on to the national economy. Although there is an extensive literature on the planning, design, construction and operations of ports and marine terminals (see, for example, Brunn, 1979; Agerschou, 1983; and Frankel, 1987) there is limited reference to the management and operations of berths in ports under congested conditions, especially in developing countries. The broad goal of research by Fararoui (1988) was to find analytical methods that help reduce the generalised social cost of port congestion to the national economy in the short-term, without any major capital investment and port expansion. The starting point is the order of servicing in-coming vessels to a berth and, specifically, the efficiency and effectiveness of the traditional scheme of "first come - first served" under congested conditions

This leads to a general definition of a port's operating objective as being to meet the need for port services at the lowest total cost to the national economy. The approach taken to achieve this general, "higher order" objective was to search for an available sequence of berthing queued vessels in a single-berth port in such a way to best satisfy a set of lower-level objectives that may be conflicting and non-commensurable. Multiple criteria decision making methods are suited to this kind of problem, and the first section of this paper classifies the various approaches before selecting multi-attribute decision making methods, with two different algorithms (SAW and TOPSIS). The second section applies both algorithms (which are explained formally in Appendix A) to a hypothetical example of priority berthing in ports. The third section expands on the objectives for the model and the weights associated with the attributes. The final section describes briefly a computer program CONPORT that has been developed on an IBM-PC.

MULTI-ATTRIBUTE DECISION MAKING FRAMEWORK

Rational decision making is defined as a systematic process for the selection of the most appropriate course of action from a set of feasible alternatives in order to utilize scarce resources in such a way to maximise the performance of the particular system under consideration. Performance may be assessed by a number of measures or attributes. In this case, multi-attribute rational decision making has six principal steps: (a) problem definition; (b) identification of objectives and their measures of attainment (attributes); (c) generation of feasible alternatives; (d) development of an evaluation model; (e) evaluation of the alternatives by means of the model; and (f) either selection of the best alternative, or ranking of the best alternatives.

Multi-attribute decision making techniques combine the outcomes (attainable attribute levels) of all alternatives for each individual attribute into a matrix, the so called "decision matrix, P" (Table 1). This task has associated with it an order of "m x n", where m is the total number of feasible alternatives and n is the number of attributes in the problem. The decision matrix, P, may be considered as a brief description of "m" alternatives which focus on "n" different attributes. Each column of matrix, "P", shows the value of the related attribute, for different alternatives, and each row represents the consequences for all attributes of a certain alternative.

Table 1: A Typical Decision Matrix for a Multi-Attribute Decision Making Problem

Alternatives	Attributes (x_j)			
A_i	x_1	x_2	x_j	x_n
A_1	p_{11}	p_{12}	p_{1j}	p_{1n}
A_2	p_{21}	p_{22}	p_{2j}	p_{2n}
.		.		.
A_i	p_{i1}	p_{i2}	p_{ij}	p_{in}
.				.
A_m	p_{m1}	p_{m2}	p_{mj}	p_{mn}

Having expressed the problem in the form of a decision matrix, then, the amount of information in this matrix, should possibly be reduced, normally in stages, in such a way to lead to the selection of a unique alternative as the best solution. There are different methods that may be used to achieve this, most of which use information concerning the relative importance of the various Pij-scores (ie. priorities, or weights). There are three terms which are, irrespective of the method used, common to all decision making techniques: non-dominated solutions; preferred solutions; and optimal solutions.

Multiple-criteria decision making (MCDM) refers to "making decisions in the presence of multiple, usually conflicting, objectives" (Zionts, 1983, p.85). Such problems may be broadly classified into multiple-objective decision making (MODM), where an infinite number of alternatives are defined implicitly by a set of constraints, and into multiple-attribute decision making (MADM), where the task is preference ranking of a finite set of decision alternatives, each of which is explicitly described in terms of different attributes. As shown in Figure 1. multi-attribute decision making methods are further classified into non-compensatory models (where there is no trade-off between attributes and the final selection is made on an attribute by attribute basis) and compensatory models, that do permit a trade-off between attributes, and, at their final step, assign a single number to each multi-dimensional objective which is then used as the main selection indicator for the best solution. Based on the means of calculating this final number, Hwang and Yoon (1981) have divided compensatory models into scoring models, compromising models and concordance models. As noted in Figure 1, these three models each have different mechanisms for calculating the final solution.

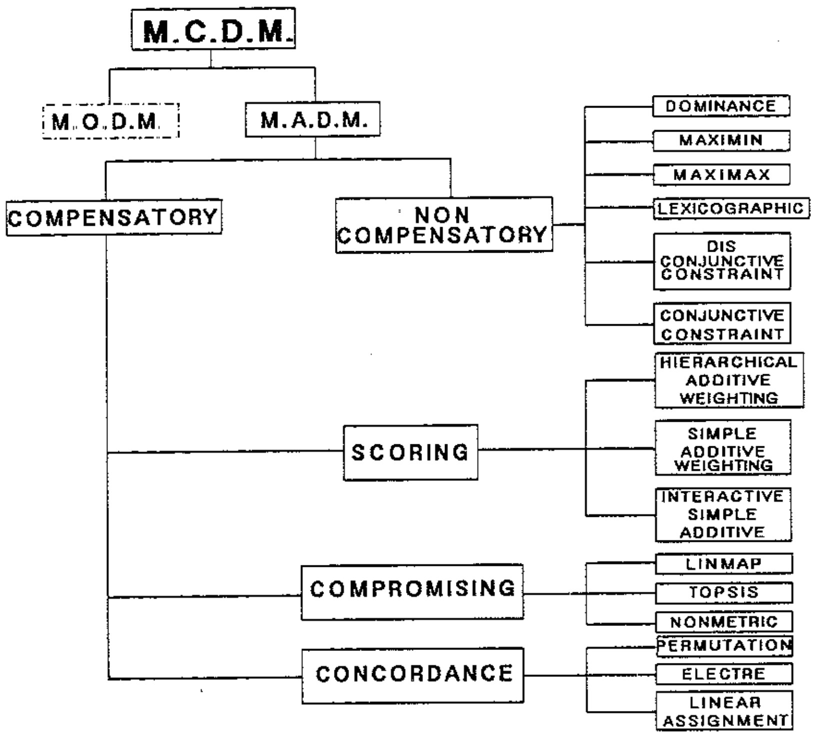

Figure 1: Classification of Multiple-Criteria Decision Making (MCDM) Methods into Multiple-Objective Decision Making (MODM) and Multiple-Attribute Decision Making (MADM).

Research by Fararoui (1988) selected an example of a scoring model - using a simple additive weighting (SAW) method; a compromising model - using the technique for order preference by similarity to ideal solution (TOPSIS); and a concordance model - using "elimination et choix traduisant la realite" (ELECTRE). In selecting these three methods, attention was paid to type of data available, time required to introduce data functions into a computer program that was to be developed, CPU time on the computer, and applicability to real world problems. Each method was applied to the problem of berthing in a congested port using a simple worked example (see next section) to test their appropriateness but ELECTRE (Benayoun, et al, 1966) did not always lead to a single, preferred solution for the sequence of berthing, and hence was not developed further.

PRIORITY BERTHING IN PORTS - A MADM EXAMPLE

The general problem can be stated as follows. If the traditional scheme of "first come - first served" is adopted for serving the vessels in a single berth port, then the system is "determined" and no improvement may be expected. However, when this restriction is relaxed, with "S" vessels, S! permutation sequencings are possible. In addition, these "m = S!" alternatives (A_i, i = 1, 2, ..., m) have been measured with data for "n" attributes (x_j, j = 1, 2, ..., n). To consider a specific numerical example, four attributes (n = 4), and three vessels (m = 3! =6) are specified for this problem. Here the six alternatives represent the ordering of vessels labelled 1, 2 and 3. The attributes set out in Table 2 are as follow:

- x_1 = to minimise cost of delay to vessels;
- x_2 = to minimise cost of delay to cargo on board;
- x_3 = to minimise total cost of delay to vessels plus cargo on board; and
- x_4 = to minimise diverse effects of the cargo delayed on board of the vessels, on national projects (because this may be quantified by combining the tonnage of each cargo-type on board and an urgency index of this cargo it leads to an attribute whose value is to be maximised).

A literature review has revealed that attribute x_4 is especially difficult to quantify (see, for example, Drewry, 1977, p. 123).

This problem is now stated in the decision matrix, "P", (Table 2), wherethe vessels are numbered 1,2, and 3 - rather than by the actual name - in the order of arrival. The first alternative (1 2 3) is the order of "first come - first served". This table demonstrates that no single alternative can best satisfy all four objectives simulataneously. Indeed, in this artificial example, the first alternative, which is the sequence of first come - first served, does not satisfy any of the four objectives.

Table 2: Decision Matrix for Sequence of Three Vessels into a Berth

Alternatives		Attributes (x_j)			
No.	A_i	x_1	x_2	x_3	x_4
1	1 2 3	301,400	152,900	454,300	614,000
2	1 3 2	275,000	575,000	332,500	571,000
3	2 1 3	100,200*	150,100	250,300	629,000
4	2 3 1	156,500	85,000*	241,500*	715,000
5	3 1 2	326,700	129,200	455,900	730,000
6	3 2 1	395,000	195,100	590,100	773,000*

* The best attribute values

The algorithm for the simple additive weighting (SAW) method is explained in Appendix A. Step 1 is obtained from Table 2. Step 2 is the normalisation of this matrix (Table 3).

Table 3: The Normalised Decision Matrix, SAW Algorithm

Alternatives No.	A_i	x_1	x_2	x_3	x_4
1	1 2 3	0.332	0.376	0.532	0.794
2	1 3 2	0.364	1.000	0.726	0.739
3	2 1 3	1.000*	0.383	0.965	0.814
4	2 3 1	0.640	0.676	1.000*	0.925
5	3 1 2	0.307	0.445	0.530	0.944
6	3 2 1	0.254	0.295	0.409	1.000*

For step 3, we assume that a decision maker assigns a set of individual weights to the four attributes, x_1, x_2, x_3, x_4, as follows: $W = \{0.17, 0.33, 0.25, 0.25\}$. The weighted normalised decision matrix is obtained by multiplying each attribute value in Table 3 by its attached weight. For example, $W_{11} = 0.332 \times 0.17 = 0.056$, and so on. The weighted normalised decision matrix for the problem is given in Table 4.

Table 4: Weighted Normalised Decision Matrix, SAW Algorithm

Alternatives No.	A_i	x_1	x_2	x_3	x_4
1	1 2 3	0.056	0.124	0.133	0.198
2	1 3 2	0.062	0.330	0.181	0.185
3	2 1 3	0.170	0.126	0.241	0.203
4	2 3 1	0.109	0.223	0.250	0.231
5	3 1 2	0.052	0.147	0.132	0.236
6	3 2 1	0.043	0.097	0.102	0.250

Finally, the weighted values of the alternatives (Step 4) are: A (1 2 3) = 0.511; A (1 3 2) = 0.758; A (2 1 3) = 0.740; A (2 3 1) = 0.813; A (3 1 2) = 0.567; A (3 2 1) = 0.492. According to the "weighted average value" of these alternatives,the best sequence of serving the vessels is the order in alternative number four - that is (2 3 1) which gives the highest of 0.813

The same problem is now solved using the TOPSIS method. The first steps of the TOPSIS algorithm (Appendix A) lead to the same weighted normalised decision index matrix given in Table 4 However, for the TOPSIS method, the ideal solution and the negative ideal solution are identified as the best and worst values for each attribute (Table 5).

Table 5: Ideal and Negative-Ideal Solutions, TOPSIS Algorithm

Extreme solutions		x_1	x_2	x_3	x_4
Ideal solution	A*	0.170	0.330	0.250	0.250
Negative-ideal solution	A-	0.043	0.097	0.102	0.185

The values of the separation measure are as follows:

$d_1^* = 0.268$ $\quad$ $d_1^- = 0.045$
$d_2^* = 0.144$ $\quad$ $d_2^- = 0.257$
$d_3^* = 0.209$ $\quad$ $d_3^- = 0.191$
$d_4^* = 0.125$ $\quad$ $d_4^- = 0.210$
$d_5^* = 0.248$ $\quad$ $d_5^- = 0.078$
$d_6^* = 0.304$ $\quad$ $d_6^- = 0.065$

Finally, the relative closeness to the ideal solution is calculated:

$C_1^* = 0.144$
$C_2^* = 0.632$
$C_3^* = 0.477$
$C_4^* = 0.627$
$C_5^* = 0.239$
$C_6^* = 0.176$

The best alternative course of action is selected based on the highest value of the C_i^*, in this case alternative number 2 or a sequence of vessels (1 3 2). Alternatively, the solutions may preferentially be ranked as follows: (1 3 2), (2 3 1), (2 1 3), (3 1 2), (3 2 1), (1 2 3). It is noted that SAW and TOPSIS give rise to slightly different rankings of alternatives, but both methods led to a single preferred solution (unlike the ELECTRE method).

MODEL DEVELOPMENT

One of the major problems in the establishment of a multi-attribute decision making model for ports, arises from the fact that no two ports, even when they are in the same country, operate under the same conditions, and, consequently, it is difficult to specify a general set of objectives that are suitable to all ports. However, improvement of ship turnaround time (in view of the enormous capital commitment now involved in shipping) has generally been accepted to be a common objective for port operators and port planners (Sinclaire, 1977; Bennathan and Walters, 1979). The cost of delay to ships occur during both idle time and active time, and it varies directly as a function of the "cost of ship's time", which itself varies for different ship type and ship size: large ships cost more per day or per hour than do small ships. There is also a counter argument, which is based on the economics of ship size: smaller ships carry less cargo but the relationship between this and their daily cost is such that the daily cost per ton of cargo is higher than for large ships (Goss, 1974). Drewry (1977, p.39) expresses yet another opinion on the cost analysis of vessels: "a large ship is cheaper at sea but more expensive in port per ton of cargo".

Of these three perspectives, the first approach is based on the cost of the vessels, whereas the later two emphasise the overall "cost per ton of cargo". To avoid any confusion in the development of the model between the cost of vessels and "cost per ton of cargo", three objectives are defined: to miminise the cost of delay to vessels; to minimise the cost of delay to cargo on board of vessels; and to minimise the cost of delay to vessels plus cargo on board.

Studies of congested ports have revealed that there have been occasions where goods have been needed urgently for vital national development projects. When these types of cargo are delayed, and not delivered on time, developmental projects to which they are linked are delayed and problems escalate. Another objective of a system of priority berthing is "to minimise the indirect costs of delay to cargo on board of vessels". This system consists of classification of all those groups on the prioriy rating of cargo (for example, 1 = highest priority), and then assigning an index to each vessel, based on the tonnage and priority ranking of each type of cargo.

The main idea behind weight assignment is to make sure that the country in question absorbs the maximum benefit from the implementation of the "systerm of priority berthing". In the assignment of such weights, the benefits are divided into two broad categories: time saving benefit to vessels; and time saving benefits to cargo on board of the vessels. In order to convert "the time saving benefits of the vessels" to the "benefit to the national economy", a few assumptions have to be made: the benefit of reducing delay to vessels will not be fully absorbed by the national economy - only half of the value of such benefit will be transferred to the country in question (Agerschou, et al, 1983, p.20) and all cargo is paid for by the consignees at the time of loading in the port of origin - thus, time saving benefits of cargo will be fully absorbed by the country. Coupling these assumptions with the objectives of the system of prioriy berthing, it is now possible to assign the relative weights fo $w_1 = 0.5$, $w_2 = 1.0$, and $w_3 = 0.75$ to the three attributes x_1, x_2, and x_3. In assigning a weight to the fourth attribute (cargo urgency index maximisation), an assumption is made that all of the indirect benefits of reducing delay to cargo are totally absorbed by the country in question. Hence, this objective receives a weight of one ($w_4 = 1$). These weights may now be normalised to sum to 1: ($w_1 = 0.15$; $w_2 = 0.31$; $w_3 = 0.23$; and $w_4 = 0.31$).

Consider a port with a single, multi-purpose berth, having "n" distinct ships in the queue awaiting berthing. These ships can be placed in a list for service according to their order of berthing, in n! different ways. The optimization method consists of testing each possible sequence of serving vessels against all other alternatives. With "n" vessels, n! permutation sequencing are possible, and when the number of ships increases, the manual generation of the alternatives, and calculation of the attribute values, will become more difficult and time consuming (for example, when the number of ships in a queue increases from 3 to 4, the number of alternatives increases from $3! = 6$ to $4! = 24$). Therefore, a computer program is highly desirable for the optimisation process.

CONPORT COMPUTER PROGRAM

A computer model, called CONPORT, has been developed on an IBM personal computer for optimising generalised social cost of the congested ports, by searching through all the possible sequence of berthings, and selecting the best alternative. Each ship in the model is known by a number from 1 to n, rather than by her real name. To enable the decision maker to compare each alternative working order, against the traditional scheme of the "first come - first served", the ships receive the numbers which reflect their order of registration in the queue of the vessels waiting to be worked.

As input to the model, it is assumed that the service time for each type of vessel, including its cargo handling characteristics, is known to the port operator based on historical data, and current productivity rates of cargo handling equipment. However, if such information is unavailable or the details are invalid for current, and/or projected, operations, then there is a need to resort to simulation methods to estimate service times.

Although time for any ship can be costed by the model, provided the ship type and deadweight tonnage/container capacity is specified, the model generates time costs of the following ship types: bulk carriers, container vessels, general cargo vessels, LASH vessels, oil/bulk/ore carriers, roll-on roll-off vessels, and tankers for a typical range of values for deadweight tonnage/container capacities (Fararoui, 1988, Table 5.3, p.139). After generating each alternative sequence of berthing, CONPORT calculates all the attribute values for the objectives. Options exist within the model to allow the user to change any of the following four parameters: interest rate; maximum number of cargo urgency classifications; price of new containers; and the associated weight of each objective.

CONPORT is designed to enable the user to examine the best alternative sequence of berthing, according to the two different multiple-attribute decision making (MADM) methods: "simple additive weighting" (SAW), and TOPSIS (Figure 2).

CONPORT
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Solution Menu

0. Exit to Main Menu
1. show PLAN IMPACT MATRIX, P
2. show alternatives with at least one extreme point
3. Show NORMALIZED DECISION MATRIX, R
4. Show FINAL DECISION MATRIX WITH "SAW" METHOD
5. Show FINAL DECISION MATRIX WITH "TOPSIS" METHOD

Enter selection

Figure 2: Format of the Solution Menu for CONPORT

The program has been designed to be "user-friendly" and easy to use. Hence, every attempt has been made to avoid entering the data which are irrelevant to the purpose of the optimisation. The following data are required to run the program: (a) the number of vessels (or a specified deviation from the expected service time); and (b) for each vessel - type of vessel, size of vessel, number of containers on board, category of cargo (1 = highest urgency), tonnage of each category of cargo, value of each category of cargo, and estimated service time of each vessel.

The model has been developed for ports with one single, multi-purpose berth and therefore any port, throughout the world, operating under this condition could use CONPORT. Under congested conditions, we would envisage that a port operator responsible for berth allocation to vessels could fruitfully run the model with data normally available to port operators and customs and find the best sequencing of berthing, given the weightings implicit in the program. The cost of ship's time, which is in the data file, would require updating to cover technological changes and the price of new ships. However, attribute weights may have a temporal dimensions as national circumstance and trade alter, and their determination should be the subject of investigation by a multidisciplinary team of national planners.

CONCLUSIONS

The purpose of the research was to develop a decision-making model for a single berth, general purpose port operating under congested conditions. Costs of additional vessel turnaround time, and costs of delay to cargo on board, are normally passed on to the national economy. Under such conditions, a port's operating objective may be broadly defined as meeting the need for port services at the lowest possible total cost to the national economy. The approach taken in the research was to examine the efficiency of the traditional scheme of "first come - first served" and to search for an available sequence of berthing the vessels in a queue in such a way as to best satisfy a set of lower order objectives.

A range of multiple criteria decision making methods were reviewed and multi-attribute decision making methods were selected as being suitable for the basis of developing the decision-making model. Two slightly different algorithms - SAW and TOPSIS - are given in the Appendix and evaluated with a numerical example that contained four attributes: cost of delay to vessels; cost of delay to cargo on board; total cost of delay to vessel and cargo; and priority of cargo for national development projects. Further developments of the model in terms of port objectives and the main concept of weight assignment were explained. Finally, a microcomputer program called CONPORT, which is designed to enable the user to examine the best possible sequence of berthing, according to the two different multi-attribute decision making methods - SAW and TOPSIS - was outlined.

ACKNOWLEDGEMENTS

Mr Farzad Fararoui would like to thank the State Rail Authority of New South Wales for their support in allowing this research to be undertaken. The authors acknowledge the helpful comments made by an anonymous referee.

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APPENDIX A - ALGORITHMS

Algorithm of the Simple Additive Weighting (SAW) Method

The SAW algorithm involves four consecutive steps:

1. Construct the decision matrix, as in Table 1 of the main body of the paper.

2. Standardise the decision matrix.
Normalisation aims at obtaining comparable scales within the attribute values. Hence, the decision maker makes a numerical scaling of intra-attribute values in order to reflect his or her marginal worth assessment within each attribute. Among different methods of normalisation, the "linear scale transformation" has been selected (Table A-1).

Table A-1: Vector Normalisation Formulae

Criteria	All Cost	All Benefit	Cost and Benefit: No of C.C. > B.C.	Cost and Benefit: No of B.C.> C.C.
x_{ij}: (cost criterion)	$r_{ij} = 1 - \frac{x_{ij}}{x_j^{max}}$	—	$r_{ij} = \frac{x_j^{min}}{x_{ij}}$	$r_{ij} = 1 - \frac{x_{ij}}{x_j^{max}}$
x_{ij}: (benefit criterion)	—	$r_{ij} = \frac{x_{ij}}{x_j^{max}}$	$r_{ij} = \frac{x_{ij}}{x_j^{max}}$	$r_{ij} = 1 - \frac{x_j^{min}}{x_{ij}}$

C.C. = Cost Criteria
B.C. = Benefit Criteria

3. Assign a weight to each attribute.
Although a variety of techniques and methods have been developed for an assessment of the weights, in fact, the decision maker can imagine his or her own technique in the context of the particular application and situation. Irrespective of the technique used for the weight assessment, the weights are usually normalised to sum to unity:

$$\sum_{j=1}^{n} w_j = 1$$

4. Obtain a total score for each alternative course of action by multiplying the normalised value of each attribute (r_{ij}) by its weight (w_j) and then summing these products over all attributes.

The simple additive weighting method may, mathematically, be represented as;

$$U(x_i) = \sum_{j=1}^{n} w_j \cdot r_{ij}$$

where, the preferred solution for the problem, A*, is:

$$A^{*} = \left\{ A_i \left| \max \sum_{j=1}^{n} w_j \quad r_{ij} \right| \right\}$$

The term "$w_j \cdot r_{ij}$" is called a marginal utility function (Fandel and Spronk, 1983). When the utility function is additive, the trade off ratio between two criteria are independent of the value of n-2 other criteria. This form of independence is called preferential independence, which is a necessary condition for normalisation.

<u>TOPSIS Algorithm</u>

The TOPSIS algorithm may be simplified into seven successive steps.

1. Construct the decision matrix, as in Table 1 of the main body of the paper.

2. Standardise the decision matrix.

   Normalisation aims to transform the various attribute dimensions into nondimentional attributes, or attributes with comparable scale. The normalised element, r_{ij}, of the normalised decision matrix R, may be calculated as;

$$r_{ij} = \frac{x_j^{min}}{x_{ij}} \quad \text{for cost criterion}$$

$$r_{ij} = \frac{x_{ij}}{x_j^{max}} \quad \text{for benefit criterion}$$

   All the criteria, costs and benefits, which are normalised by this method, will be converted to the benefit criteria, ie. the greater the value of the attribute, the more the benefit.

Table A-2: Normalised Decision Matrix in TOPSIS Algorithm

Alternatives A_i	Attributes (x_j) x_1	x_2	...	x_j	...	x_n
A_1	r_{11}	r_{12}	...	r_{1j}	...	r_{1n}
A_2	r_{21}	r_{22}	...	r_{2j}	...	r_{2n}
...	...	...		...		...
A_i	r_{i1}	r_{i2}	...	r_{ij}	...	r_{in}
.	...	...		...		
A_m	r_{m1}	r_{m2}	...	r_{mj}	...	r_{mn}

3. Construct the weighted normalised decision matrix.
Since all the criteria in a multi-attribute decision making problem are not of the same importance, each criterion receives a weight from the decision maker. These weights are usually normalised to sum to 1. The weighted normalised decision matrix, v, is constructed by multiplying each attribute value, r_{ij}, by its weight, w_j.
Therefore,

$$v_{ij} = r_{ij} \cdot w_j$$

4. Determine the ideal and negative-ideal solutions.
The ideal solution, or the most preferred solution, is the solution with the best attribute values for all the criteria, i.e. a maximum value for the benefit criterion and a minimum value for the cost criterion (v_j^*):

$$A^* = \left\{ v_1^*, v_2^*, \ldots v_n^* \right\}$$

and the negative-ideal solution, A^-, or the least preferred solution with the worst attribute value for all criterion, i.e. a maximum attribute value for cost criteria, and a minimum attribute value for benefit criteria (v_j^-);

$$A^- = \left\{ v_1^-, v_2^-, \ldots v_n^- \right\}$$

5. Calculate the separation measure.
The distance of each alternative solution from the ideal and from the negative-ideal solutions may be calculated as its Euclidean distance from these solutions, respectively:

$$d_i^* = \left[\sum_{j=1}^{n} \left(v_{ij} - v_j^* \right)^2 \right]^{\frac{1}{2}} \quad i = 1, 2, \ldots, m$$

and

$$d_i^- = \left[\sum_{j=1}^{n} \left(v_{ij} - v_j^- \right)^2 \right]^{\frac{1}{2}} \quad i = 1, 2, \ldots m$$

where, d_i^* and d_i^- are the separation distance of alternative solution "i" from the ideal and negative-ideal solutions, respectively.

6. Calculate the relative closeness to the ideal solution.

The relative closeness of alternative A_i to the ideal solution A^* is calculated as:

$$C_i^* = \frac{d_i^-}{\left(d_i^* + d_i^-\right)} \quad 0 < C_i^* < 1 \quad i = 1, 2, \ldots, m$$

where C_i^* is the relative closeness of alternative A_i to the ideal solution. Clearly, the relative closeness of the ideal solution A^* to itself is one ($C_*^* = 1$) and that of the negative-ideal solution to the ideal solution is zero ($C_-^* = 0$). Hence, an alternative solution A_i is preferable to alternative A_k if $C_i^* > C_k^*$.

7. Select the best alternative.
The alternative with the maximum relative closeness to the ideal solution is the preferred solution. Alternatively, one may preferentially rank the solutions according to their relative closeness.