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# FORECASTING THE EFFECTS OF CHANGES IN THE WORK-PLACE LOCATIONS ON RESIDENTIAL DEVELOPMENT AND TRIP GENERATION

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*ABSTRACT:*

*In this paper, we consider the problem of forecasting the development of residential sections of an urban area in the near future (5 to 10 years hence). A special entropy model is developed which gives the most probable residential development for given changes in work-place. A simple algorithm is devised which efficiently solves the resulting non-linear mathematical program. This analysis is applied to a sample city. The results provide estimates of the changes in residential locations as well as imputed trip origin and destination costs. Trip distribution is carried out implicity.*

## FORECASTING RESIDENTIAL DEVELOPMENT

### INTRODUCTION

One commonly used approach in urban planning today involves the preparation and evaluation of several alternative strategies for the urban area under study. Each strategy represents a physical development plan for land use and facilities in the city, and the evaluation attempts to determine that strategy with lowest total cost. This cost includes such variables as infrastructure cost, travel time, environmental, pollution and social costs, etc.

The Planning and Environmental Commission is currently preparing alternative strategy plans for the Hunter Region, and one aspect of this exercise that lends itself to modelling is the following. Given that there are alternative locations for future industrial and service employment areas in the region, how can one forecast the probable development of future residential areas, corresponding to each alternative distribution of jobs? An answer to this question would ensure that the planner's recommendations for future work-place locations and residential areas would be consistent with each other. Although there are many mathematical urban models that would appear to be relevant to this problem, it has been found necessary to modify existing models to take into account the infringement of zonal population capacities.

Entropy models as defined by Wilson (1970) have contributed significantly to explaining residential location patterns in urban regions and of inter-urban traffic flow. Residential location models are often referred to as attraction-constrained models while trip generation transport models are cited as production-attraction-constrained models. Both concepts are interpreted as members of a family of spatial interaction models in urban processes.

To build such models we firstly assume that our study area is divided into zones which are numbered sequentially. We define the level of interaction between zone i and zone j as $T_{ij}$. In particular, this interaction will represent the flow of workers from residences to jobs and hence the system under investigation can be thought of as assignments of individual workers to an origin-destination table.

For our purposes $T_{ij}$ is the number of home-work trips for zone i to zone j. To determine the $T_{ij}$ we are given the following information:

$O_i$ the total number of workers who live in i

$D_j$ the total number of jobs in j

$c_{ij}$ the 'cost' of travelling from i to j

$C$ the total expenditure on travel to work

$T$ the total number of workers to be allocated

There are many sets of $T_{ij}$ which would satisfy the given data. It makes sense therefore to choose that set of $T_{ij}$ which is most probable. This is the "Principle of Insufficient Reason" of Laplace. The resulting problem is the maximization of entropy given the data available. We have the following mathematical model.

Model A Maximize

$$\frac{T!}{\prod_{i,j} T_{ij}!} \tag{1}$$

$$\sum_j T_{ij} = O_i, \quad \forall\, i \tag{2}$$

$$\sum_i T_{ij} = D_j, \quad \forall\, j \tag{3}$$

$$\sum_{i,j} c_{ij}\, T_{ij} = C \tag{4}$$

$$T_{ij} \geq 0 \qquad \forall\, i,j$$

Model A is, of course, the doubly constrained "gravity" model mentioned earlier and is inappropriate for a residential location model since the $O_i$'s are to be determined rather than given. It is, however, appropriate as a transport model. The usual attraction-constrained model is also not appropriate because it does not impose a housing capacity constraint on each zone, although Batty (1976) has shown how this model may be modified to take account of this constraint. For this reason, Model B has been formulated to impose this constraint explicitly and it is this modification which makes Model B more suitable for residential location than earlier models used.

In Model B the housing capacity $O_i$ for zone i is replaced by the parameter $G_i$ which is the potential housing capacity in zone i in existing or proposed accommodation. The equality constraints (2) are replaced by inequality constraints (6) and we obtain the following mathematical problem:

Model B Maximize $$\frac{T!}{\prod_{i,j} T_{ij}!} \tag{5}$$

subject to $$\sum_j T_{ij} \le G_i \,, \quad \forall\ i \tag{6}$$

$$\sum_i T_{ij} = D_j \,, \quad \forall\ j \tag{7}$$

$$\sum_{i,j} c_{ij}\ T_{ij} = C \tag{8}$$

$$T_{ij} \ge 0 \qquad \forall\ i,j$$

In the next section each model is analysed via geometric programming and algorithms for their solution are presented. Then Model B is applied to both a model city and the Newcastle (N.S.W.) urban area. Finally, conclusions on this analysis are presented.

METHODOLOGY

In this section we take the two models presented in the introduction and develop them. The models are first analysed then algorithms for solution are presented. To use the models calibration is next considered. Finally forecasting using the entropy models is developed.

Analysis of the Entropy Models

To facilitate the analysis two changes are required. The factorials in the objective functions are cumbersome. These shall be replaced by somewhat simpler functions using the Stirling approximation ($N! = N^N\ e^{-N}$). The budget constraints, (4) and (8), shall be absorbed into the objective function (1) and (5) respectively by the factor $\exp\{-\beta(\sum_{i,j} c_{ij}\ T_{ij} - C)\}$. $\beta$ is a multiplier associated with the equality (4) or (8) and since the maximization is not over $\beta$ or C, the term $\beta C$ may be dropped. Let $R_{ij} = e^{-\beta c_{ij}}$. Model A now becomes:

Model C Maximize $$\prod_{i,j} \left(\frac{R_{ij}}{T_{ij}}\right)^{T_{ij}} (T)^T \tag{9}$$

subject to $\sum_j T_{ij} = O_i \ , \quad \forall\ i$ (10)

$\sum_i T_{ij} = D_j \ , \quad \forall\ j$ (11)

$T_{ij} \geq 0 \ , \quad \forall\ i,j$

Model B becomes:

Model D Maximize $\prod_{i,j} \left(\frac{R_{ij}}{T_{ij}}\right)^{T_{ij}} (T)^{T}$ (12)

subject to $\sum_j T_{ij} \leq G_i \ , \quad \forall\ i$ (13)

$\sum_i T_{ij} = D_j \ , \quad \forall\ j$ (14)

$T_{ij} \geq 0 \ , \quad \forall\ i,j$

A dual problem can be obtained for each of the models C and D. The dual to Model C is:

Model E Minimize $\prod_i q_i^{-O_i} \prod_j p_j^{-D_j}$ (15)

subject to $\sum_{i,j} R_{ij}\, q_i\, p_j \leq 1$ (16)

$q_i > 0 \ , \quad \forall\ i$

$p_j > 0 \ , \quad \forall\ j$

The dual to Model D is:

Model F Minimize $\prod_i q_i^{-G_i} \prod_j p_j^{-D_j}$ (17)

subject to $\sum_{i,j} R_{ij}\, q_i\, p_j \leq 1$ (18)

$0 < q_i \leq 1 \ , \quad \forall\ i$

$p_j > 0 \ , \quad \forall\ j$

These duals are developed using geometric programming in Jefferson and Scott (1978).

When these models C, D, E, F are solved the relationship between the solutions of the pairs C,E and D,F is:

$$\frac{T_{ij}}{T} = R_{ij}\, q_i\, p_j \;, \quad \forall\; i,j \tag{19}$$

$R_{ij}/(\sum_{i,j} R_{ij})$ is the a priori probability of a trip going from i to j. $R_{ij}\, q_i\, p_j$ is the a posteriori probability of a trip going from i to j. Relating the $R_{ij}$ back to travel costs $c_{ij}$ we recall $\beta c_{ij} = -\log R_{ij}$. We can associate a new travel cost $\hat{c}_{ij}$ with the a posteriori probability by

$$\hat{c}_{ij} = -\frac{\log R_{ij}}{\beta} - \frac{\log q_i}{\beta} - \frac{\log p_j}{\beta} \tag{20}$$

Defining
$$c_{i0} = -\frac{\log q_i}{\beta}$$
$$c_{0j} = -\frac{\log p_j}{\beta}$$

(20) becomes

$$\hat{c}_{ij} = c_{ij} + c_{i0} + c_{0j}$$

$c_{i0}$ is the cost associated with the equation

$$\sum_j T_{ij} = O_i \;, \forall\; i \quad \text{or inequality} \sum_j T_{ij} \le G_i \;, \forall\; i.$$

$c_{0j}$ is the cost associated with the equation

$$\sum_i T_{ij} = D_j \;, \forall\; j$$

The costs $c_{i0}$, $c_{0j}$ may be positive or negative.

Note that the entropy models affect the cost $c_{ij}$ only through costs associated with origin ($c_{i0}$) or destination ($c_{0j}$). Congestion costs along the route from i to j are not captured. If congestion costs differ from the original estimates they must be calculated separately. The focus on the near future should alleviate the need for their estimation.

Algorithms

To solve the models C and D, algorithms can be devised through the analysis of the pairs of models C,E and D,F (see Jefferson and Scott, 1978). To solve C,E we use the following algorithm:

Step 1 Set $n = 0$ and $p_j^n = D_j/T$

Step 2 Set n to n + 1 and

$$q_i^n = \frac{O_i}{T(\sum_j R_{ij}\, p_j^{n-1})}$$

Step 3

$$p_j^n = \frac{D_j}{T(\sum_i R_{ij}\, q_i^n)}$$

Step 4 Feasibility:

$$T\, q_i^n \sum_j R_{ij}\, p_j^n = O_i + e_i^n , \quad \forall\, i$$

where $e_i^n$ is the error in the $i^{th}$ constraint. If $|e_i^n|$ is less than a predetermined tolerance for all i go to step 5. Otherwise go to step 6.

Step 5 Optimality:

$$d^n = \sum_i e_i^n \log q_i^n$$

is the difference between the logarithms of the objective functions for C and D. $|d^n|$ should be less than a prescribed tolerance. If so go to step 7. Otherwise go to step 6.

Step 6 Set n to n + 1 and

$$p_j^n = p_j^{n-1} / \sum_j p_j^{n-1}$$

Go to step 2.

Step 7 Set $T_{ij} = T R_{ij} q_i^n p_j^n$.

For the pair D,F the algorithm changes slightly to the following algorithm:

Step 1 Set $n = 0$ and $q_i^n = 1$

Step 2 Set

$$p_j^n = \frac{D_j}{T(\sum_i R_{ij} q_i^n)}$$

Step 3 Check for feasibility:

$$T q_i^n \sum_j R_{ij} p_j^n \le G_i + e_i^n , \quad \forall i$$

where $e_i^n \ge 0$ is the error on the $i^{th}$ inequality. If $e_i^n$ is less than a predetermined tolerance for all i go to step 4. Otherwise go to step 5.

Step 4 Check for optimality.

$$d^n = \sum_i e_i^n \log q_i^n$$

is the difference between the logarithms of the objective functions for D and F. $|d^n|$ should be less than a pre-described tolerance. If so go to step 6. Otherwise go to step 5.

Step 5 Set n to n + 1

$$q_i^n = \min \left\{ 1, \frac{G_i}{T(\sum_j R_{ij} p_j^{n-1})} \right\}$$

Go to step 2.

Step 6 Set

$$T_{ij} = T R_{ij} q_i^n p_j^n$$

Both of the algorithms require only the updating of the q and p vectors. The computation involves little more than the calculation of the inner product of q and p with the appropriate row or column of the R matrix which remains constant throughout the computations.

## Calibration and Forecasting

To use the entropy models we require estimates for the first model of $O_i$, $D_j$ and $R_{ij}$. The $O_i$ and $D_j$ are straightforward. If one knows the $T_{ij}$ then a good estimate of the $R_{ij}$ is

$$R_{ij} = T_{ij}/T$$

However knowledge of the $T_{ij}$ is often hard to come by. The costs of travel from i to j ($c_{ij}$) are easier to calculate. $R_{ij}$ and $c_{ij}$ are related by

$$R_{ij} = 1 \Big/ \frac{1}{(e^{c_{ij}})^{\beta}}$$

where $\beta$ must be determined. If some $T_{ij}$ or aggregates of $T_{ij}$ are known (such as traffic flow along an arterial road), a choice of $\beta$ which minimizes the sums of squares of $(T_{ij} - T\,R_{ij})$ would be reasonable. $\beta$ could also be obtained from a previous gravity model of the city under consideration or of a city with similar characteristics.

To obtain forecasts from the second model of future demand we need to know the parameters $G_i$, $D_j$, and $R_{ij}$ for this model. Under the assumption of little or no change in the $c_{ij}$'s and $\beta$ a good estimate of $R_{ij}$ would be

$$R_{ij} = T_{ij}/T$$

where $T_{ij}$ and T are obtained from solution of model C. The $D_j$ is calculated from forecasts of jobs in the business and financial districts of the city in the future. The $G_i$ measures the numbers of houses or other sorts of domiciles possible in a particular zone. We assume that all jobs will be filled but all available land is not necessarily built on.

The solution to model D is the most probable distribution of trips from the various origins to the various destinations.

## ANALYSIS OF A SAMPLE CITY

The residential location model, model B, has been programmed via the algorithm and results obtained for both a small model city and the Newcastle Urban Area.

### Model City

The data for a 12 zone model city was extracted from a paper by Dacey and Norcliffe (1976). No comparison of results with those of their model will be presented. Table 1 gives the travel cost matrix ($c_{ij}$). The algorithm was programmed on a PDP 11/45 mini-computer using RSTS/E FORTRAN with a maximum job core capacity of about 26K. The program was fully core resident. The total number of workers to be allocated residences was T = 200,000.

Table 2 provides output when $\beta$ was chosen as 0.80 with specified tolerances $|d^n|$ = .01% and $e_i$ = 1% × $G_i$. The number of iterations required for convergence was 23, representing a CPU time of 7 seconds.

TABLE 1

Travel Cost Matrix ($c_{ij}$) for Model City

| Residential | Employment Zone j | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Zone i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1 | 1.2 | 2.4 | 2.8 | 4.6 | 2.4 | 3.8 | 5.6 | 4.0 | 6.0 | 7.7 | 6.7 | 7.7 |
| 2 | 2.4 | 1.0 | 3.6 | 3.2 | 4.8 | 6.2 | 7.6 | 6.4 | 6.1 | 5.6 | 4.3 | 5.3 |
| 3 | 2.8 | 3.6 | 2.0 | 3.0 | 3.0 | 4.4 | 4.7 | 3.2 | 3.2 | 4.9 | 5.1 | 6.0 |
| 4 | 4.6 | 3.2 | 3.0 | 1.0 | 6.0 | 7.4 | 7.7 | 6.1 | 3.9 | 3.3 | 2.1 | 3.1 |
| 5 | 2.4 | 4.8 | 3.0 | 6.0 | 0.8 | 1.4 | 2.8 | 1.6 | 3.8 | 5.2 | 5.6 | 6.6 |
| 6 | 3.8 | 6.2 | 4.4 | 7.4 | 1.4 | 0.5 | 1.4 | 3.0 | 5.2 | 3.1 | 4.4 | 5.4 |
| 7 | 5.6 | 7.6 | 4.7 | 7.7 | 2.8 | 1.4 | 0.6 | 1.5 | 3.4 | 1.7 | 3.0 | 4.0 |
| 8 | 4.0 | 6.4 | 3.2 | 6.1 | 1.6 | 3.0 | 1.5 | 0.8 | 2.2 | 3.2 | 4.0 | 5.0 |
| 9 | 6.0 | 6.1 | 3.2 | 3.9 | 3.8 | 5.2 | 3.4 | 2.2 | 1.0 | 1.7 | 1.8 | 2.8 |
| 10 | 7.7 | 5.6 | 4.9 | 3.3 | 5.2 | 3.1 | 1.7 | 3.2 | 1.7 | 0.9 | 1.3 | 2.3 |
| 11 | 6.7 | 4.3 | 5.1 | 2.1 | 5.6 | 4.4 | 3.0 | 4.0 | 1.8 | 1.3 | 0.6 | 1.0 |
| 12 | 7.7 | 5.3 | 6.0 | 3.1 | 6.6 | 5.4 | 4.0 | 5.0 | 2.8 | 2.3 | 1.0 | .05 |

TABLE 2

Output for Model City for $\beta = 0.8$

| Residential Zone $i$ | Employment Zone $j$ ($T_{ij}$) 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | Predicted Workers $\sum_j T_{ij}$ | Maximum Workers $G_i$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 13256 | 2174 | 3877 | 174 | 353 | 38 | 7 | 98 | 14 | 1 | 4 | 1 | 20000 | 20000 |
| 2 | 7025 | 9224 | 2830 | 740 | 72 | 8 | 2 | 20 | 18 | 11 | 42 | 8 | 20000 | 20000 |
| 3 | 1687 | 381 | 3365 | 287 | 100 | 11 | 7 | 85 | 62 | 6 | 7 | 2 | 6000 | 6000 |
| 4 | 2973 | 2903 | 11248 | 10581 | 67 | 7 | 5 | 62 | 264 | 172 | 602 | 116 | 30000 | 30000 |
| 5 | 10946 | 687 | 7125 | 123 | 2734 | 555 | 151 | 1446 | 181 | 24 | 23 | 4 | 24000 | 24000 |
| 6 | 5851 | 368 | 3809 | 66 | 2772 | 1869 | 758 | 773 | 97 | 209 | 99 | 19 | 16691 | 20000 |
| 7 | 1386 | 120 | 2996 | 52 | 905 | 910 | 1438 | 2566 | 409 | 641 | 304 | 59 | 11785 | 20000 |
| 8 | 4986 | 313 | 9948 | 186 | 2362 | 253 | 700 | 4493 | 1067 | 193 | 137 | 26 | 24664 | 40000 |
| 9 | 1007 | 398 | 9948 | 1079 | 406 | 44 | 153 | 1466 | 2787 | 641 | 794 | 153 | 18877 | 20000 |
| 10 | 235 | 540 | 2323 | 1587 | 121 | 212 | 542 | 599 | 1448 | 1106 | 1078 | 208 | 10000 | 10000 |
| 11 | 389 | 1136 | 1470 | 3078 | 65 | 56 | 142 | 235 | 993 | 597 | 1402 | 437 | 10000 | 10000 |
| 12 | 258 | 755 | 1059 | 2047 | 43 | 37 | 95 | 156 | 660 | 397 | 1506 | 966 | 7980 | 20000 |
| Available jobs $\sum_i T_{ij}$ | 50000 | 20000 | 60000 | 20000 | 10000 | 4000 | 4000 | 12000 | 8000 | 4000 | 6000 | 2000 | | |
| | | | | | | | | | | | | Total | 199997 | |

Newcastle Urban Area

The second application of the algorithm was to the Newcastle Urban Area with a population of around 340,000. The region is divided into 77 district traffic zones and the $T_{ij}$'s were determined from a travel time matrix $(c_{ij})$ prepared by the Urban Transport Study Group. Due to core limitations the now larger matrices $(c_{ij})$, $(R_{ij})$ and $(T_{ij})$ were accessed from secondary storage (RK05 disks).

Table 3 presents output for a run with β chosen as 0.08, which represents an average travel to work time of approximately 17 minutes, being close to the value calculated from survey data. Tolerances were set at $e_i$ = 1 and $|d^n|$ = .01. The number of iterations for convergence to this accuracy was 25 which represented a CPU time of approximately 2 minutes and a total elapsed time of 3½ minutes.

TABLE 3

Output for Newcastle Urban Area for β = 0.08

| Zone | Given Employment | Given Population | Simulated Population | Increase | Maximum Population |
|---|---|---|---|---|---|
| 1 | 13818 | 3297 | 3369 | +72 | 3369 |
| 2 | 8292 | 801 | 835 | +34 | 835 |
| 3 | 4895 | 1604 | 1634 | +30 | 1634 |
| 4 | 2132 | 6354 | 6401 | +47 | 6401 |
| 5 | 1319 | 5705 | 5904 | +199 | 5904 |
| 6 | 2955 | 4357 | 4412 | +55 | 4412 |
| 7 | 3089 | 5191 | 5241 | +50 | 5241 |
| 8 | 755 | 6436 | 6484 | +48 | 6484 |
| 9 | 161 | 3656 | 3836 | +180 | 3836 |
| 10 | 916 | 6889 | 6977 | +88 | 6977 |
| 11 | 1340 | 6511 | 6702 | +191 | 6702 |
| 12 | 1137 | 5900 | 5820 | -80 | 5933 |
| 13 | 7483 | 2122 | 2122 | +0 | 2122 |
| 14 | 3502 | 7020 | 7051 | +31 | 7051 |
| 15 | 2051 | 8734 | 8923 | +189 | 8923 |
| 16 | 14788 | 749 | 749 | -0 | 749 |
| 17 | 4851 | 7096 | 7186 | +90 | 7186 |

TABLE 3 (Cont'd)

| Zone | Given Employment | Given Population | Simulated Population | Increase | Maximum Population |
|---|---|---|---|---|---|
| 18 | 551 | 39 | 39 | +0 | 39 |
| 19 | 3271 | 2229 | 2229 | +0 | 2229 |
| 20 | 2759 | 6307 | 6367 | +60 | 6367 |
| 21 | 2027 | 7937 | 8379 | +442 | 8379 |
| 22 | 926 | 6980 | 7521 | +541 | 7521 |
| 23 | 853 | 6724 | 7067 | +343 | 7067 |
| 24 | 1704 | 6082 | 6535 | +453 | 6535 |
| 25 | 969 | 0 | 0 | 0 | 0 |
| 26 | 1921 | 3033 | 3033 | +0 | 3033 |
| 27 | 1180 | 7546 | 7923 | +377 | 7923 |
| 28 | 832 | 6452 | 9396 | +2944 | 9443 |
| 29 | 1050 | 4524 | 9150 | +4626 | 12937 |
| 30 | 262 | 468 | 6242 | +5774 | 6242 |
| 31 | 380 | 390 | 390 | +0 | 390 |
| 32 | 1799 | 4876 | 6620 | +1744 | 17779 |
| 33 | 475 | 5869 | 6788 | +919 | 6788 |
| 34 | 884 | 4110 | 4768 | +658 | 4768 |
| 35 | 323 | 7827 | 6463 | -1364 | 15039 |
| 36 | 1416 | 5204 | 4332 | -872 | 9528 |
| 37 | 341 | 3120 | 4662 | +1542 | 5789 |
| 38 | 548 | 6536 | 5201 | -1335 | 14508 |
| 39 | 395 | 4177 | 4607 | +430 | 10835 |
| 40 | 2015 | 6935 | 3872 | -3063 | 11808 |
| 41 | 160 | 6235 | 2454 | -3781 | 6837 |
| 42 | 554 | 6222 | 1730 | -4492 | 6726 |
| 43 | 337 | 859 | 1046 | +187 | 1046 |
| 44 | 662 | 6055 | 7220 | +1165 | 7220 |
| 45 | 835 | 6530 | 8375 | +1845 | 9567 |
| 46 | 1444 | 5891 | 8375 | +2484 | 11252 |
| 47 | 664 | 5637 | 6849 | +1212 | 12820 |
| 48 | 2315 | 4869 | 6035 | +1166 | 6582 |

TABLE 3 (Cont'd)

| Zone | Given Employment | Given Population | Simulated Population | Increase | Maximum Population |
|---|---|---|---|---|---|
| 49 | 850 | 5438 | 7592 | +2154 | 11912 |
| 50 | 813 | 5711 | 4374 | -1337 | 34633 |
| 51 | 128 | 272 | 295 | +23 | 295 |
| 52 | 794 | 5386 | 5062 | -324 | 15904 |
| 53 | 1250 | 8687 | 3867 | -4820 | 15156 |
| 54 | 673 | 2690 | 2521 | -169 | 5363 |
| 55 | 168 | 775 | 2803 | +2028 | 23910 |
| 56 | 256 | 447 | 481 | +34 | 481 |
| 57 | 1072 | 6037 | 2098 | -3939 | 7678 |
| 58 | 690 | 537 | 560 | +23 | 560 |
| 59 | 4340 | 2249 | 2249 | +0 | 2249 |
| 60 | 131 | 714 | 714 | 0 | 714 |
| 61 | 1085 | 6086 | 5451 | -635 | 13535 |
| 62 | 734 | 2671 | 6319 | +3648 | 23723 |
| 63 | 1478 | 5626 | 5390 | -236 | 38647 |
| 64 | 1021 | 6432 | 4793 | -1639 | 12764 |
| 65 | 514 | 3030 | 3343 | +313 | 11623 |
| 66 | 3356 | 4189 | 4189 | +0 | 4189 |
| 67 | 2988 | 7329 | 4350 | -2979 | 25714 |
| 68 | 68 | 611 | 4338 | +3727 | 90872 |
| 69 | 471 | 618 | 618 | +0 | 618 |
| 70 | 1577 | 7040 | 4908 | -2132 | 7040 |
| 71 | 593 | 4867 | 3967 | -900 | 4867 |
| 72 | 241 | 239 | 239 | 0 | 239 |
| 73 | 2754 | 7318 | 4102 | -3216 | 7318 |
| 74 | 2384 | 8842 | 3988 | -4854 | 8842 |
| 75 | 242 | 797 | 797 | 0 | 797 |
| 76 | 24 | 297 | 297 | +0 | 297 |
| 77 | 220 | 330 | 330 | +0 | 330 |
| Totals | 138250 | 337350 | 337350 | | 691027 |

TABLE 4

Travel Costs Associated with the Origin ($c_{i0}$) and Destination ($c_{0j}$)

| Zone | $c_{i0}$ | $c_{0j}$ | Zone | $c_{i0}$ | $c_{0j}$ | Zone | $c_{i0}$ | $c_{0j}$ |
|---|---|---|---|---|---|---|---|---|
| 1 | -18.46 | 18.23 | 27 | -18.65 | 18.71 | 53 | -6.41 | 15.15 |
| 2 | -18.71 | 18.25 | 28 | -22.70 | 18.68 | 54 | -14.90 | 13.98 |
| 3 | -18.42 | 18.26 | 29 | -26.76 | 18.66 | 55 | -32.06 | 15.59 |
| 4 | -18.27 | 18.22 | 30 | -49.93 | 18.32 | 56 | -15.92 | 13.71 |
| 5 | -18.60 | 18.21 | 31 | -17.98 | 18.32 | 57 | 0 | 8.96 |
| 6 | -18.33 | 18.32 | 32 | -21.48 | 18.27 | 58 | -12.78 | 10.19 |
| 7 | -18.28 | 18.29 | 33 | -19.79 | 17.58 | 59 | -18.06 | 18.13 |
| 8 | -18.26 | 18.13 | 34 | -19.75 | 17.50 | 60 | -17.99 | 18.11 |
| 9 | -18.74 | 18.06 | 35 | -15.40 | 16.94 | 61 | -16.44 | 17.91 |
| 10 | -18.28 | 18.16 | 36 | -15.29 | 16.07 | 62 | -28.25 | 17.99 |
| 11 | -18.42 | 18.01 | 37 | -22.42 | 15.83 | 63 | -16.36 | 16.60 |
| 12 | -17.99 | 18.32 | 38 | -14.68 | 16.37 | 64 | -13.19 | 16.65 |
| 13 | -18.19 | 18.30 | 39 | -18.57 | 16.14 | 65 | -17.54 | 16.05 |
| 14 | -18.24 | 18.34 | 40 | -9.76 | 14.60 | 66 | -16.35 | 15.99 |
| 15 | -18.45 | 18.40 | 41 | -4.98 | 12.01 | 67 | -9.62 | 15.35 |
| 16 | -18.19 | 18.37 | 42 | -.01 | 9.83 | 68 | -40.36 | 15.00 |
| 17 | -18.31 | 18.27 | 43 | -18.02 | 7.07 | 69 | -14.89 | 13.45 |
| 18 | -18.15 | 18.41 | 44 | -20.15 | 18.04 | 70 | -11.13 | 14.64 |
| 19 | -18.17 | 18.46 | 45 | -21.01 | 18.22 | 71 | -11.11 | 11.55 |
| 20 | -18.28 | 18.46 | 46 | -22.18 | 17.94 | 72 | -15.70 | 14.75 |
| 21 | -18.82 | 18.53 | 47 | -20.09 | 17.32 | 73 | -4.10 | 8.25 |
| 22 | -19.03 | 18.30 | 48 | -20.15 | 17.23 | 74 | -1.22 | 7.95 |
| 23 | -18.72 | 18.66 | 49 | -21.84 | 17.98 | 75 | -11.22 | 8.74 |
| 24 | -19.03 | 18.59 | 50 | -13.85 | 17.16 | 76 | -15.06 | 14.05 |
| 25 | - | 18.72 | 51 | -18.28 | 17.20 | 77 | -16.92 | 16.90 |
| 26 | -18.05 | 18.57 | 52 | -16.43 | 16.68 | | | |

The entropy model considers what would happen if jobs were frozen and residential construction was allowed to continue. This plan is more to test the model rather than to consider a conceivable future. As can be seen from table 3 the model indicates some movement of the domiciles Areas of increase are attractive; areas of decrease unattractive. The planner thus is concerned with possible overcrowding in areas of increase and with deterioration in areas of decrease.

Of greater interest is the information provided by table 4. The table provides imputed changes in costs at the origin and destination. The $c_{i0}$ is negative and represents a drop in the costs of accommodation. The $c_{0j}$ is positive and represents a drop in wages. Both $c_{i0}$ and $c_{0j}$ decrease in absolute value as one moves out from the CBD. The changes in costs infer a loss in momentum of the city's economy.

While the $c_{0j}$ (the costs associated with the workplace) are relatively consistent there is more variation in the $c_{i0}$. The cost changes associated with residential location ($c_{i0}$) are abnormally high in zones 30, 55 and 68 indicating that people would prefer to live in other zones when accommodation becomes available. The cost change is abnormally low in the following zones: 35, 36, 38, 40, 41, 42, 50, 52, 53, 54, 56, 57, 58, 61, 63, 64, 66, 67, 69, 70, 71, 73 and 74. These zones have better than average attraction.

These results do concur with knowledge about the Newcastle area.

Finally as a by-product of the residential forecasts the entropy model provides us with the $T_{ij}$ matrix.

## CONCLUSIONS

The application of geometric programming techniques has shown that explicit incorporation of a zonal housing capacity constraint (6) to form model B, has produced a fast and efficient algorithm which avoids the pitfalls of attraction-constrained gravity models. Indeed, it would be interesting to compare the characteristics of this algorithm with those modifications suggested by Batty (1976) on the residential location gravity model.

Preliminary examination of the Newcastle Urban Area has shown it will be possible for forecasts of future residential demand in the area to be made, with a continuing study providing insight for the planner in his evaluation of alternative strategies. The preparation and refinement of alternative zonal work-force locations and future maximum household constraints is at present underway.

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